1. (a) \( P(A) \) is true for \( A = \emptyset \), \( A = S15 \), \( A = S25 \), \( A = S1, 2, 25 \)
(b) \( P(A) \) is false for \( A = S15, S1, 4, S2, 45, \) or \( S1, 2, 45 \)
(c) If \( A = \emptyset \) or \( A = S15 \), then \( A \cap S1, 2, 35 = \emptyset \);
for all other \( A \in S \), \( A \cap S1, 2, 35 \neq \emptyset \).

2. (a) \( P(A) \land Q(A) \) is true for \( A = S17, S35, S51, S1, 35, S1, 53, S3, 53, S1, 3, 53 \); for all other
\( A \in S \), \( P(A) \land Q(A) \) is false.
(b) \( P(A) \lor \neg Q(A) \) is true for all \( A \in S(3, 3, 53) \)
(c) \( P(A) \Rightarrow Q(A) \) is false for all \( A \in S \)
such that \( A \cap 32, 4, 65 = \emptyset \) and \( A = \emptyset \),
so \( P(A) \Rightarrow Q(A) \) is false when \( A = \emptyset \)
and is true for all \( A \in S - \{\emptyset\} \).

3. (a) If \( \sqrt{2} \) is rational and \( \sqrt{3} \) is rational, then
\( \sqrt{3} \) is rational.
Since \( P \) is false we have \( P \land Q \) false and
so \( (P \land Q) \Rightarrow R \) is true.

(b) If \( \sqrt{2} \) is irrational or \( \sqrt{3} \) is irrational, then \( \sqrt{3} \) is irrational.
Since \( \neg P \) is true, the statement \( (\neg P) \lor Q \)
is true. Since \( R \) is false, the statement
\( (\neg P) \lor Q \Rightarrow R \) is false.
4. \[
\begin{array}{c|c|c|c|c}
 P & Q & P \rightarrow Q & P \land (P \rightarrow Q) & (P \land (P \rightarrow Q)) \rightarrow Q \\
 T & T & T & T & T \\
 T & F & F & F & T \\
 F & T & T & F & T \\
 F & F & T & F & T \\
\end{array}
\]

Since all entries in the last column are \( T \),
the statement is a tautology.

5. \[
\begin{array}{c|c|c|c|c|c|c}
 P & Q & \sim P & \sim Q & \sim (P \lor Q) & (\sim P) \land (\sim Q) \\
 T & T & F & F & F & F \\
 T & F & F & T & F & F \\
 F & T & F & T & F & F \\
 F & F & T & T & T & T \\
\end{array}
\]

The truth table above shows that \( \sim (P \lor Q) = (\sim P) \land (\sim Q) \).

\[
\begin{array}{c|c|c|c|c|c|c}
 P & Q & \sim P & \sim Q & \sim (P \lor Q) & (\sim P) \lor (\sim Q) \\
 T & T & F & F & F & F \\
 T & F & F & T & T & T \\
 F & T & F & T & T & T \\
 F & F & T & T & T & T \\
\end{array}
\]

The truth table above shows that \( \sim (P \land Q) = (\sim P) \lor (\sim Q) \).
6. (a) Since $P(x)$ is true for all $x \neq 1$, $P(x) \Rightarrow Q(x)$ is true for $x \neq 1$. When $x = 1$, $P(x)$ is true and $Q(x)$ is false, and so $P(x) \Rightarrow Q(x)$ is false for $x = 1$.

\[ \therefore \text{The set on which } P(x) \Rightarrow Q(x) \text{ is true is } \mathbb{R} - \{1\}. \]

(b) $P(x) \Rightarrow Q(x)$ is false when $x^2 \geq 1$ and $x < 1$.

This occurs when $x \in (-\infty, -1] \cup [1, \infty) \cap (-\infty, 1) = (-\infty, -1] \cup [1, \infty) \cap (-\infty, 1) = (-\infty, -1] \cup \emptyset = (-\infty, -1]$

Thus, $P(x) \Rightarrow Q(x)$ is true for all $x \in \mathbb{R} - (-\infty, -1] = (-1, \infty)$.

(c) $P(x) \Rightarrow Q(x)$ is false when $x \in [-1, 2]$ and $x^2 < 2$.

Noting that the domain of $x$ is $S = [-1, 1]$, we have that $P(x) \Rightarrow Q(x)$ is false for $x \in [-1, 1] \cap [1, -\sqrt{2}) \cup (\sqrt{2}, 10] = \emptyset$.

\[ \therefore P(x) \Rightarrow Q(x) \text{ is true for all } x \in [-1, 2]. \]
7. (a) Since \( \pm \sqrt{2} \) are the only solutions

to \( x^2 - 2 = 0 \) and \( \pm \sqrt{2} \) are irrational,
the statement is false.

(b) False.

Reason: Given any \( x \in \mathbb{Z} \) there exist (many) \( y \in \mathbb{Z} \)
such \( x+y \neq 1 \).

(c) True. For \( x \in \mathbb{Z} \), choose \( y = 1-x \). Since \( 1-x \in \mathbb{R} \)
and \( x + (1-x) = 1 \), the statement is true.

(d) True. Choose \( x = 3 \), \( y = 0 \).

8. (a) For all \( -1 < x < 1 \) there exists a natural
number \( y \) such that \( |x+y| < 1 \).

\[ \exists x \in (-1,1) \forall y \in \mathbb{N}, \ |x+y| < 1 \]

\( P \) is false: When \( x = \frac{1}{2} \), \( |x+y| \geq 1 \) for every \( y \in \mathbb{N} \)

(b) There exists \( -1 < x < 1 \) and there exists a natural
number \( y \) such that \( |x+y| < 1 \).

\[ \forall x \in (-1,1), \forall y \in \mathbb{N}, \ |x+y| < 1 \]

\( P \) is true: pick \( x = \frac{1}{2} \) and \( y = 1 \).
(c) (i) For all $-1 < x < 1$, there exists $y \in \mathbb{R}$ such that
if $|x + y| < 1$, then $y \geq 1$.

(ii) There exists $x \in (-1, 1)$ such that for all $y \in \mathbb{R}$,
$|x + y| < 1$ and $y < 1$.

(iii) Note that if $y \in \mathbb{R}$, $y < 1$ and so $|x + y| < 1$ and $y < 1$
for every $y \in \mathbb{R}$ (and $x \in (-1, 1)$). It follows
that the statement in (ii) is false and so the original
statement is true.

(d) (i) For all $x \in (-1, 1)$ there exists $y \in \mathbb{R}$ such that if
$x < 1$, then $y < 0$.

(ii) $\exists x \in (-1, 1)$ such that for all $y \in \mathbb{R}$, $x < 1$ and $y \geq 0$

(iii) Letting $x = 0$ we note that for any $y \in \mathbb{R}$, the
statement $x^2 < 1$ and $y \geq 0$ is true and so the
statement in (ii) is true. It follows that our original
statement is false.