Homework Assignment # 4

Math 481
Kaul
Fall 2018
Due Tuesday, October 23

Instructions: To receive full credit, each solution must be neat and legible. Explain your reasoning fully and use complete sentences when appropriate – an answer without an explanation will receive no credit. Staple the homework sheet to the front of your work.

For problems 1-3, no justification is needed – just right down the answer.

1. Fraleigh p. 85 # 23
2. Fraleigh p. 85 # 24
3. Fraleigh p. 85 # 26

4. Let \( h : \mathbb{Z}_5 \rightarrow S_{\mathbb{Z}_5} \) be the left-regular representation of \( \mathbb{Z}_5 \). For each \( n \in \mathbb{Z}_5 \), compute \( h(1)(n) \) and \( h(3)(n) \).

5. Let \( G \) be a group and let \( a \in G \). The set \( C(a) = \{ g \in G \mid ga = ag \} \) is called the centralizer of \( a \).
   (a) Prove that \( C(a) \leq G \).
   (b) Use the Cayley table for \( S_3 \) given on p. 79 to find the centralizer of \( \rho_1 = (1, 2, 3) \).
   (c) Use the Cayley table for \( D_4 \) given on p. 80 to find the centralizer of \( \mu_1 = (1, 2)(3, 4) \).

6. Fraleigh p. 94 # 12
7. Fraleigh p. 96 # 29
8. Fraleigh p. 96 # 31
10. Fraleigh p. 102 # 16

11. Let \( G \) be a group and let \( H \leq G \). Prove: If \( g^{-1}hg \in H \) for all \( g \in G \) and all \( h \in H \), then \( gH = Hg \) for all \( g \in G \).

12. Recall the subgroup \( U = \{ z \in \mathbb{C}^* \mid |z| = 1 \} \) of \( \mathbb{C}^* \)
   (a) Prove or disprove: \( (1 + 2i)U = (2 + i)U \).
   (b) Prove or disprove: \( (1 + 2i) = 3U \).
   (c) Describe the elements of \( \mathbb{C}^*/U \) geometrically.

13. Prove: If \( |G| = n \) and \( g \in G \), then \( g^n = e \).