1. Show that $1 - 5 - 10 - 15 - 14 - 18 - 14 - 12 - 12$ is a valid Hilbert function (i.e., show that there is an ideal in $R[x_1, \ldots, x_5]$ such that $H(R/I) = 1 - 5 - 10 - 15 - 14 - 18 - 14 - 12 - 12$).

2. Compute the Hilbert function of $R[x, y, z]/\langle x^2 + xy + xz, xy + y^2, xz + z^2, y^3, z^3 \rangle$.

3. Show that if $H(R[x_1, \ldots, x_n]/I, d) \leq d$ for some $d \in \mathbb{N}$, then
   \[ H(R[x_1, \ldots, x_n]/I, t + 1) \leq H(R[x_1, \ldots, x_n]/I, t) \]
   for all $t \geq d$.

4. Give an example of two ideal $I$ and $J$ in $\mathbb{R}[x, y]$ such that $R/I \not\cong R/J$ but $H(R/I) = H(R/J) = 1 - 2 - 1$.

5. Suppose that $I = \langle f_1, \ldots, f_m \rangle$ is a homogeneous ideal of $R[x_1, \ldots, x_n]$ and the degree of $f_i$ equals $d_i$ for $i = 1, \ldots, m$ (and each $d_i > 1$). Show that $H(R/I, d_i) < H(R/I, d_i - 1)^{(d_i - 1)}$ for $i = 1, \ldots, m$. 