1. Prove (as we did in class) that $x, y, z$ is a primitive Pythagorean triple if and only if there are $s, t \in \mathbb{N}_{>0}$ such that $s > t$, $\gcd(s, t) = 1$, and one of $s$ and $t$ is even while the other is odd.

2. Show that $3, 4, 5$ is the smallest Pythagorean triple in the sense that if $x', y', z'$ is another triple, then $x \leq x'$, $y \leq y'$ and $z \leq z'$.

3. Show that $1, \frac{1}{2} + \frac{\sqrt{3}}{2}i$, and $\frac{1}{2} - \frac{\sqrt{3}}{2}i$ are exactly the three cube roots of unity in $\mathbb{C}$ (you can use that a polynomial has exactly the number of its degree zeros over the complex numbers).

4. Prove (as in class or by another method) that there does not exist $x, y, z \in \mathbb{N}_{>0}$ such that $x^4 + y^4 = z^2$ (you should not, of course, use Fermat’s last theorem).

5. Let $(x, y)$ be a rational point (so both $x$ and $y$ are rational) in the first quadrant on the unit circle about the origin and which does not lie on either axis. Let $L$ be the line connecting $(x, y)$ and $(-1, 0)$. Prove that the $y$-intercept of $L$ is rational.