Forces/Moments on Slider-Crank During Movement

\[ \text{FBD} = \text{MA}\overline{D} \]

\[ \sum F_x : F_{\text{rot}} = m\alpha_n = mw^2r_{\text{c/o}} \]

\[ \sum M_0 : O = 0 \quad (\text{There are no forces nor moments about O.}) \]

So, pivot force rotates around with link, always pulling it back toward point O.
Let's look at a slider:

\[ \text{Free Body Diagram (FBD)} = \quad \text{net force} \]

\[ F_y = \text{ma} \]

\[ +2F_y: \quad F_y = ma \]

So if we know \( a \) from kinematics, we should be able to find \( F_y \).

But actual piston has other forces acting on it too, namely in the \( x \)-direction. There is no movement in the \( x \)-direction, so \( 2F_x = 0 \) is a statics problem.
Real situation - Power stroke - pressure in cylinder pushes piston down, exerts torque through crank onto wheels:

\[ F_{BD} = MA \]

Assume at first no friction

\[ F_{cy} - y \rightarrow \Sigma F_x: \quad F_{cy} - F_{rx} = 0 \]

\[ +\Sigma F_y: \quad -F_p + F_{ry} = -ma \]

\( F_{ry} \) come from rod.

Question is, is rod a 2-force member? For if it is, then we know its direction, and

\( F_{rx} \) & \( F_{ry} \) are not independent.

Let’s approach this by first treating the rod as a 2-force member, then looking at it and changing that
assumption if necessary. This allows an incremental approach and also allows us to compare the two approaches and see exactly what the effect is if the rod isn’t a 2-force member. It may not be a 2-force member because high accelerations may cause movement of its X to generate forces at the ends.

\[ \text{FBD} = \text{MA} \]

\[ \begin{align*}
  \sum F_x: & \quad F_{y e} + F_r \cos \theta_2 = 0 \\
  \text{if } \theta_2 > 90^\circ, \cos \theta_2 < 0
\end{align*} \]

\[ \begin{align*}
  +\sum F_y: & \quad -F_p + F_r \sin \theta_2 = -ma
\end{align*} \]
If known $F_p$ & $a$ from kinematics, should be able to find $F_r$ & $F_{eye}$, which'll be needed to design rod & piston too.

Example:

$P = 600 \text{ bar}$

$r_2 = 150 \text{ mm}$

$r_3 = 320 \text{ mm}$

$d_{eye} = 100 \text{ mm}$

$w_2 = 5000 \text{ rpm}$

Piston @ TDC, $m = 1 \text{ kg}$

\[ \vec{a}_B = \vec{a}_A + \vec{a}_{B/A-n} + \vec{a}_{B/A-t} \]

\[ \vec{a}_A = w_2^2 r_2 \frac{1}{4} \]

\[ \vec{a}_{B/A-n} = w_3^2 r_3 \frac{1}{4} \]

\[ \vec{a}_{B/A-t} = \vec{a}_{AB} r_3 \]

From previous studies, at TDC $w_3$ has reached its maximum CW rotation & is starting now to slow down. So at TDC $\psi_3 = 0$. 
\[ \omega_{3} = \frac{V_{A}}{r_{3}} \quad \text{CW} = \frac{w_{2}r_{2}}{r_{3}} \quad \text{CW} \]

(A + TDC B is IC of link 3.)

\[ \alpha_{3} = \left( \frac{w_{2}r_{2}}{r_{3}} \right)^{2} \quad \text{\textbullet} \]

Thus \[ \alpha_{A} = \left[ \frac{w_{2}^{2}r_{2}}{1} + \left( \frac{w_{2}r_{2}}{r_{3}} \right)^{2} \right] \quad \alpha_{3} \]

\[ \alpha_{A} = w_{2}^{2}r_{2} \left( 1 + \frac{r_{2}}{r_{3}} \right) \quad \alpha_{3} \]

\[ \alpha_{A} = \left( \frac{50.00 \text{ rev}}{\text{min}} \times \frac{2 \pi \text{ rad}}{\text{rev}} \right)^{2} \quad \left[ \frac{0.150 \text{ m}}{\text{rev}} \right] \quad \left( 1 + \frac{150}{320} \right) \]

\[ \alpha_{A} = 60.4 \times 10^{3} \quad \text{m/} \text{sec}^{2} \]

\[ \Rightarrow 2F_{x} : \quad F_{xy} + F_{R} \cos 90^\circ = 0 \]

\[ F_{xy} = 0 \]

\[ \Rightarrow 2F_{y} : \quad -F_{p} + F_{R} \sin 90^\circ = -ma_{\parallel} \]

\[ F_{p} = P \cdot \frac{d^{2}}{4} \quad \Pi = 60 \text{ bar} \quad 0.1 \times 10^{6} \quad \text{Pa} \quad \text{N} \]

\[ \frac{1}{1} \quad \text{bar} \quad \frac{1}{1} \quad \text{Pa} \quad \text{m}^2 \]

\[ \left( 0.1 \text{ m} \right)^{2} \quad \Pi = 47.1 \times 10^{3} \text{ N} \]
\[ F_r = F_p - ma_b \]

\[ F_r = 47.1E3 \text{ N} - 1 \text{ kg} \times 60.4E3 \text{ m/sec}^2 \]

\[ F_r = -13.3E3 \text{ N} \]

The negative sign in the above equation means that all of the force to pull the piston down does not have to come from the rod. The pressure in the cylinder actually helps to push the piston down.

So for the case of TDC for a 4-cycle engine, a worse case is at the end of the exhaust stroke, when \( F_p = 8 \text{ bar (atmospheric)} \). Then all turning-around force on the piston must come from the rod.

\[ F_r = 60.4E3 \text{ N} \]

more than 4X at beginning of power stroke.