The duration of this test is 90 minutes. In giving your answer, the answer alone is not enough. Good engineering practice requires that you clearly give your rationale for arriving at your answer. Your pathway to your solution must be checkable. Make sure to show all work clearly when performing calculations.

Problem values:

1. The above drawing shows the cycloidal curve generated by point \( X \) on the rim of a rolling wheel as it rolls to the right. Assume the wheel is rotating at a constant rate of 1 rad/sec. The wheel's diameter is 1 m.

a. Referring to path coordinates, where is/are the point(s) where the acceleration perpendicular to the path (normal acceleration) is greatest? What is the value of this acceleration?

Max normal acceleration at 4. \( a_n = v^2 \frac{r}{r} \) is from the wheel center, whose acceleration is 0. 

\[ a_n = \left( \frac{\text{rad}}{\text{sec}^2} \right) \cdot \frac{0.5 \text{m}}{\text{sec}} = 0.5 \text{ m/sec}^2 \]

b. Again, referring to the path coordinates, where is/are the point(s) where the acceleration along the path (tangential acceleration) is greatest? What is the value of this acceleration?

At 1, the normal acceleration relative to the wheel center is tangential to the path. So max \( a_t \) is at 1, is along the path, is \( a_n = 0.5 \text{ m/sec}^2 \)

c. Between what two points on the path is the tangential acceleration pointed in the same direction as the velocity?

Between pts 1 & 4 the tangential acceleration in along the path. \( v \) is increasing.

d. Between what two points on the path is the tangential acceleration pointed against the velocity?

Between pts 4 & 7 the tangential acceleration in against \( v \)'s direction.

e. Between what two points on the path is the path-referenced normal acceleration oriented in the direction of the velocity? Explain.

Nowhere, \( a_n \) is always \( \perp \) the path. \( \vec{v} \) is always along the path, so \( a_n \perp \vec{v} \) always.
2. The figure at right shows a wheel rolling to the right without slipping on the surface, i.e., at a steady speed. Its rotation rate is $1 \text{ rad/sec}$. The wheel's diameter is $2 \text{ m}$. Answer the following questions about the wheel's motion.

a. What is $\vec{v}_O$? Give magnitude and then the direction in terms of the unit vectors shown.

$$\vec{v}_O = \omega \cdot \vec{r}_O = \frac{\text{rad}}{\text{sec}} \cdot 1 \text{ m}$$

$$\vec{v}_O = 1 \text{ m/sec} \ 0_x$$

b. Which point(s) of the 9 lettered undergo no acceleration? Justify your answer(s).

All points A - H have nonzero accelerations because of $\vec{w}$. But O moves steadily to the right with no change in speed or direction. Thus O is the only point with no acceleration.

c. What is $\vec{a}_{H/O}$?

$$\vec{a}_{H/O} = \omega^2 \vec{r}_O = \left( \frac{\text{rad}}{\text{sec}} \right)^2 \ 1 \text{ m} = \frac{1 \text{ m}}{\text{sec}^2}$$

$$\vec{a}_{H/O} = \frac{1 \text{ m}}{\text{sec}^2} \left( -\frac{1}{\sqrt{2}} \vec{e}_x + \frac{1}{\sqrt{2}} \vec{e}_y \right)$$

d. What is $\vec{a}_{D/C}$?

$$\vec{a}_{D/C} = \omega^2 \vec{r}_{D/C} = \frac{1 \text{ rad}}{\text{sec}^2} \ \sqrt{2} \text{ m} = \sqrt{2} \text{ m} \ \frac{\text{m}}{\text{sec}^2}$$

$$\vec{a}_{D/C} = \sqrt{2} \text{ m} \ \frac{\text{m}}{\text{sec}^2} \left( -\frac{1}{\sqrt{2}} \vec{e}_x + \frac{1}{\sqrt{2}} \vec{e}_y \right) = -1 \text{ m/sec} \vec{e}_x + 1 \text{ m/sec} \vec{e}_y$$

e. If the wheel is at this velocity but slowing down at a rate of $1 \text{ m/sec}^2$, what is the total acceleration of point C? Give the direction in terms of the unit vectors shown.

Now $\vec{v}_O$ is decreasing. Point A has no horizontal acceleration because of its contact with the ground.

Thus $\vec{a}_{c-t} = -\alpha \vec{r}_O \vec{e}_x = -\frac{\text{rad}}{\text{sec}^2} \ 2 \text{ m} \ \vec{e}_x = -2 \text{ m} \ \frac{\text{m}}{\text{sec}^2} \vec{e}_x$

$\vec{a}_{c-n} = -\frac{1 \text{ m}}{\text{sec}^2} \vec{e}_y$. So $\vec{a}_C = \vec{a}_{c-t} + \vec{a}_{c-n}$

$$\vec{a}_C = -2 \text{ m} \ \frac{\text{m}}{\text{sec}^2} \vec{e}_x - \frac{1 \text{ m}}{\text{sec}^2} \vec{e}_y$$

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3. The drawing at right shows the quick-return mechanism from Midterm 1. When we analyzed the Coriolis acceleration of link 3, we found that during the steady rotation of link 2, there are some positions where link 3 has no Coriolis acceleration.

a. What are those positions, and why is there no Coriolis acceleration at those points?

Five-term acceleration equation:

\[ \vec{a}_B = \vec{a}_A + \vec{a}_{xy} + \vec{a}_{cor} + \vec{a}_n + \vec{a}_t \]

\[ \vec{a}_{cor} = 2 \vec{\omega} \times \vec{v}_{xy} \]

At the position shown with link 2 at -30°, link 4 is at its maximum rightward position \(\theta_2\) is turning around, so \(\omega_4 = 0\). This is the rotation rate of the rotating coordinate system \(\dot{xy}\), whose origin is at \(O_2\). At the mirrored position \((\theta_2 = -150°)\), \(\omega_4\) is also 0. Since \(a_{cor} = 2 \omega V_{xy}\), \(a_{cor} = 0\) at these two points.

When \(\theta_2 = 90°\) (TDC of link 2) & \(\theta_2 = 270°\) (BDC of link 2), \(V_3 = 0\). So \(V_{xy} = 0\) here too. So at \(\theta_2 = 90°\) & \(\theta_2 = 270°\), \(V_{xy} = 0\), so \(a_{cor} = 0\). So at \(\theta_2 = -30°, 90°, -150°,\) and \(270°\), \(a_{cor} = 0\).

b. Draw on the drawing the fixed coordinate system \((X/Y)\) and the rotating coordinate system \((x/y)\) that would be logical to use to analyze the accelerations of link 3.
4. The drawing at right shows a constant acceleration cam, with its lobe extended over the entire 360° of the cam's circumference.

a. How long does it take the follower to reach its opening position (rise phase)?

\[ w = \text{1 rad/sec}, \quad \text{so 1 rev lasts } 2\pi \text{ sec.} \]

\[ t_{0-180} = \frac{180}{360} \times \frac{2\pi}{2\pi} = \frac{1}{2} \text{ sec} \]

b. How long is the follower's fall phase?

\[ t_{210-360} = 2\pi \times \frac{5}{360}/12 \]

\[ t_{210-360} = \frac{5}{6} \pi \text{ sec} \]

c. If the cam is turning at 1 rad/sec, what is the maximum velocity during the rise phase?

\[ AV_1 = \frac{1}{2} t_{0-90} V_{\text{max-rise}} \]

\[ V_{\text{max-rise}} = \frac{2AV_1}{t_{0-90}} = \frac{2 (0.5 \text{ cm})}{100 \text{ cm} \times \pi \text{ sec}} = \frac{0.02 \text{ m}}{8.14 \text{ sec}} \]

\[ V_{\text{max-rise}} = 0.02 \frac{\text{m}}{\text{sec}} \]

d. What is the cam's maximum velocity during the fall phase?

\[ V_{\text{max-fall}} = \frac{2AV_1}{t_{20-185}} = \frac{0.24 \text{ m}}{5\pi \text{ sec}} = 0.015279 \frac{\text{m}}{\text{sec}} \]

\[ V_{\text{max-fall}} = 0.00637 \frac{\text{m}}{\text{sec}} \]

e. What is the acceleration/deceleration during the rise phase?

\[ a_{\text{rise}} = \frac{AV_{\text{rise}}}{t_{\text{rise}}} = \frac{0.00637 \frac{\text{m}}{\text{sec}}}{2\pi \text{ sec}} = 0.004055 \frac{\text{m}}{\text{sec}^2} \]

f. What is the acceleration/deceleration during the fall phase?

\[ a_{\text{fall}} = \frac{AV_{\text{fall}}}{t_{\text{fall}}} = \frac{0.015279 \frac{\text{m}}{\text{sec}}}{2\pi \text{ sec}} = 0.01167 \frac{\text{m}}{\text{sec}^2} \]
5. In the mechanism at right, \( v_c = 1 \text{ m/sec} \), as shown.

a. Draw the velocity diagram for \( v_B \).

\[
\vec{v}_B = \vec{v}_C + \vec{v}_{B/C}
\]

b. Calculate \( v_B \).

\[
\begin{align*}
V_B &= V_c \cos 70 + V_{B/C} \cos 60 \\
V_c \sin 70 &= V_{B/C} \sin 60 \\
V_{B/C} &= \frac{V_c \sin 70}{\sin 60} \\
V_B &= V_c \left( \cos 70 + \frac{\sin 70}{\sin 60} \cos 60 \right) \\
V_B &= 0.885 \text{ m/sec}
\end{align*}
\]

c. Calculate \( \vec{\omega}_{BC} \).

\[
\omega_{BC} = \frac{V_{B/C}}{r_{B/C}} = \frac{V_c \sin 70}{\sin 60} \frac{1}{0.102 \text{ m}}
\]

\[
\omega_{BC} = 10.6 \text{ rad/ sec}, \text{ CCW}
\]