Slider-crank analytically

- Want position of piston as \( f(\theta) \)
- Also want \( \theta_3 \) as \( f(\theta_2) \)
- So can get \( \omega_2 \).
- Then can plot \( y_B(\theta_2) \).
- Its slope will be \( y_B'(\theta_2) \).
- Then its slope will be \( y_B''(\theta_2) \).

Likewise, if get \( \theta_2(\theta_3) \), its slope will be \( \omega_3(\theta_3) \) and its slope, in turn, will be \( \omega_3'(\theta_2) \).

Known \( \omega_2, r_2, r_3 \)

\[ r_2 \cos \theta_2 = r_3 \cos \phi_3 \]
\[ = r_3 \cos (\theta_3 - 180) \]

In this equation, all that's unknown is \( \phi_3 \)

\[ \phi_3 = \arccos \left( \frac{r_2 \cos \theta_2}{r_3} \right) \]

Then \( y_B = r_2 \sin \theta_2 + r_3 \sin \phi_3 \)

\[ \theta_3 = 180^\circ - \phi_3 \]
Velocities

\[ \dot{y}_B = r_2 \cos \theta_2 \dot{\theta}_2 + r_3 \cos \phi_3 \dot{\phi}_3 \]

Here don't know \( \dot{y}_B \) nor \( \dot{\phi}_3 \)

\[ \cos \phi_3 = \frac{r_2}{r_3} \cos \theta_2 \]

\[ \phi_3 = \arccos \frac{r_2}{r_3} \cos \theta_2 \]

\[ \frac{d \phi_3}{dt} = \frac{d \phi_3}{d \theta_2} \frac{d \theta_2}{dt} \quad \text{(chain rule)} \]

\[ \frac{d \arccos x}{dx} = -\frac{1}{\sqrt{1-x^2}} \]

\[ \phi_3 = -\frac{1}{\sqrt{1-(\frac{r_2}{r_3} \cos \theta_2)^2}} \dot{\theta}_2 \]

Accelerations

\[ \ddot{y}_B = -r_2 \sin \theta_2 \ddot{\theta}_2 + r_2 \cos \theta_2 \dot{\theta}_2^2 + \text{two term}\]

\[ -r_3 \sin \phi_3 \dot{\phi}_3^2 + r_3 \cos \phi_3 \dot{\phi}_3^2 \]

Have \( \phi_3 \) from above as \( f(\theta_2, \dot{\theta}_2) \)
\[ \phi_3 = \frac{d \phi_3}{d \theta_2} \frac{d \theta_2}{dt} \quad \text{if } \theta_2 \text{ is constant} \]

\[ \text{if } \theta_2 \text{ is not constant, } \text{then will need to} \]

\[ \text{add that effect in.} \]

\[ \frac{d \phi_3}{dt} = \frac{d \phi_3}{dx} \frac{dx}{d \theta_2} \frac{d \theta_2}{dt} \]

\[ \frac{d x}{dx} = \frac{d (x^{-1})}{dx} = -x^{-2} = -\frac{1}{x^2} \]

Here \( x = \sqrt{1 - \left(\frac{r_3}{r_2} \cos \theta_2 \right)^2} \)

So need chain rule here too: \( \frac{d \phi_3}{d \theta_2} \frac{d \theta_2}{dt} = \dot{\theta}_3 \)

\[ \frac{d \phi_3}{d \theta_2} = \frac{d \phi_3}{dx} \frac{dx}{d \theta_2} = \frac{d (x^{-1})}{dx} \frac{dx}{d \theta_2} = -\frac{1}{x^2} \frac{d x}{d \theta_2} \]

\[ \frac{d x}{d \theta_2} = \frac{1}{2} \left[ 1 - \left( \frac{r_3}{r_2} \cos \theta_2 \right)^2 \right]^{-\frac{1}{2}} \left( \frac{2 \frac{r_3}{r_2} \cos \theta_2}{\cos \theta_2} \right) \left( + \sin \theta_2 \right) \]

\[ \frac{d \phi_3}{dt} = \frac{1}{1 + \left( \frac{r_3}{r_2} \cos \theta_2 \right)^2} \frac{1}{2} \left[ 1 + \left( \frac{r_3}{r_2} \cos \theta_2 \right)^2 \right]^{-\frac{1}{2}} \]

\[ 2 \frac{r_3}{r_2} \cos \theta_2 \sin \theta_2 \dot{\theta}_2 \]

All quantities we know here.
So there's actually 2 ways to calculate $V_B$, $\alpha_B$, $W_3$, and $\alpha_3$:

1) Use the above-developed equations

2) Plot $y_B = y_B(t)$ and take its slope, to get $\dot{y}_B(t)$

Then in turn, $\dot{y}_B(t)$'s slope to get $\ddot{y}_B(t)$

Do same for $\theta_3$, $\dot{\theta}_3$, $\ddot{\theta}_3$

From above accelerations can get forces. Actually for pin forced $A$ would also need the acceleration of the mass center of link 3.

Also, another solution method directly for $V_B$ and $\alpha_B$ is to vector equations. But this method may already do that, since double dots & dots squared are already coming out of it.

Need to check above solution method for all 4 quadrants after confirming it's correct.
When $\theta_2 > 90^\circ$ (QII)

\[
\begin{align*}
\gamma_2 \cos \phi_2 &= \gamma_3 \cos \theta_3 \\
\gamma_2 \cos (180 - \theta_2) &= \gamma_3 \cos \theta_3 \\
\theta_3 &= \arccos \left( \frac{\gamma_2 \cos (180 - \theta_2)}{\gamma_3} \right)
\end{align*}
\]

Compare that with

\[
\theta_3 = 180^\circ - \arccos \left( \frac{\gamma_2}{\gamma_3} \cos \theta_2 \right)
\]

When $\theta_2 = 90^\circ$, both eq's give $\theta_3 = 90^\circ$

(Could also get piston position and link 3 orientation simply by mirroring across vertical plane, i.e.:

$\gamma_p (120) = \gamma_p (60)$

$\theta_2 = 90 + 30 \rightarrow 90 - 30 = \theta_2$

So $90$ is the reference & deviation from it are equal, so, e.g.

$y_B (180) = y_B (0) \ (\theta_{dev} = 90^\circ)$
Can we infer anything about bottom half of revolution from top half? yes I think we can. The angle of link 3 will be the same for the bottom half, mirrored about the central horizontal location, i.e. the mid-stroke travel of the piston is at $\Theta_2 = 0^\circ$ & $180^\circ$.

And e.g., $y_B(\Theta_2 = 10^\circ) = y_B(\Theta_2 = -10^\circ)$.

$y_B(170^\circ) = y_B(10^\circ)$, $y_B(180^\circ) = y_B(0^\circ)$

So in $Q\Pi$ we can get $y_B$ by mirroring across vertical position.

In $Q\Pi$ & $Q\Pi$ we can get $y_B$ & $\Theta_3$ by mirroring across horizontal position.

In Summary, left-to-right; the $y_B$'s are mirrored with the $y_B$'s equal depending on the angular displacement of link 2 from the vertical; top-to-bottom the $y_B$'s are displaced from $y_B=$middle by corresponding rotation of $\Theta_2$ from $0^\circ$ or $180^\circ$.

Do for every $5^\circ$, 72 times.
Each instance is an equal unit of time separated, if \( w_2 = \text{constant} \)
Make y_b in two parts

\[ r_3 \sin \phi_3 \quad \frac{1}{2} \frac{1}{2} \sin \theta_2 \]

and are

\[ \theta_2 = 90^\circ \]

\[ \frac{y_b}{r_3 \cos \theta_3} \]

\[ \theta_3 \]

\[ \frac{1}{2} \frac{1}{2} \sin \theta_2 \]

\[ \theta_2 = 180^\circ \]

\[ r_2 \sin \theta_2 \]

\[ r_2 \sin \theta_3 - r_2 \sin \theta_2 \]

Need to work angles out carefully.

So that when take skipes, it's all

Correct \[ \theta_3 = 90^\circ \]

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But is this automatically taken care of by sine shifts in different quadrants?

\[ y = r_2 \sin \theta_2 + r_3 \sin \theta_3 \]

will subtract 0 \( \leq \theta_2 < 180^\circ \)

0 \( \leq \theta_2 < 360^\circ \) \ 
\sin \ never +

and will mirror will mirror

around horizontal axis about 90°

We're finding \[ \theta_3 = \theta_3 (\theta_2) \]

\[ r_2 \cos \theta_2 = r_3 |\cos \theta_3| = r_3 (-\cos \theta_3) \quad \theta_2 < 90 \]

\[ = r_3 \cos \theta_3 \quad 90 < \theta_2 < 180 \]

\[ = r_3 (-\cos \theta) \quad 180 < \theta_2 < 360 \]

So \[ \theta_3 = \arccos \left( \frac{r_2 \cos \theta_2}{r_3} \right) \quad -90 < \theta_2 < 90 \]

\[ = \arccos \left( \frac{r_2 \cos \theta_2}{r_3} \right) \quad 90 < \theta_2 < 270 \]