Homework 5

Problem 5.1 – First-order step response

We want a unit step response of a first-order system with $K_{ss} = -5$ and $T = 8$ sec. We’ll let step occur at 1 sec and let the simulation run for $5 \cdot T = 41$ sec.

The model:

![First-order system](image)

Step = 1 @ 1 sec; $K_{ss} = -5, T = 8$ sec

The results:

With a step input of 1, the system moves to -5 because of $K_{ss} = -5$.

Let’s check the time constant. It should take 8 sec for the output to travel to 63.2% of its final value.

$$0.632 \times (-5) = -3.175$$

The response plot above is annotated to show these parameters on the plot. It takes about $4 \cdot T$ to go from its initial position to its final position. That would be at 33 sec on the above plot.
Problem 5.2 — Second-order step response

Produce a 2\textsuperscript{nd}-order step response with a step size of 2 starting at \( t = 1 \) sec, \( K_{ss} = 8 \), %OS = 20\%, and \( \omega_n = 1 \) Hz.

![Modelica simulation](image)

The above response was plotted from the Modelica simulation shown below:

Second-order system

Step = 2 @ 1 sec; \( K_{ss} = 8 \), \( \omega_n = 1 \) Hz

Since the step size is 2, a \( K_{ss} \) of 8 will have the output jump from 0 to 16 as a steady-state value, which it does.

In the 2\textsuperscript{nd}-order block, \( \omega_n \) was specified as 6.28 rad/sec, which is \( 2 \times \pi \) rad/sec, which gives a period of about 1 sec, as shown above. I.e. the second peak, barely perceptible, is a little over 1 sec after the first peak. What is shown above is the \( \omega_d \), the damped frequency, which is always a little bit slower than the natural frequency.

To get the 20\%OS, I could have calculated what the damping should be, because that is all that %OS depends on (see Chapter 4), but instead I just kept changing \( D (\zeta) \) and rerunning the simulation until I had about 20\%OS. \( 20 \% \) of 16 is 3.2, and you can see that the peak is at about 16+3.2 = 19.2. As you can see in the dialog box below, I got 20\%OS with \( D = 0.45 \).
The peak time, $T_p = \pi/\omega_d$, where $\omega_d$ is the damped frequency (what is shown in the response). And

$$\omega_d = \omega_n \cdot \sqrt{1 - \zeta^2} = 6.28 \cdot \sqrt{1 - 0.45^2} = 0.560$$
Problem 5.3 – PID controller

Simulate a PID controller in detail, with $K_P = 10$, $K_I = 1$, $K_D = 2$. Use a step with a value of 2 that starts at $t = 1$ sec and run the simulation for 11 seconds.

The model:

![PID controller diagram]

The response:

![Response graph]

Derivative kick happens with step, when slope of input curve is $\infty$
A close-up of the plot is shown below:

Step size of 2 with KP = 10, starts curve off at 20

With KI = 1 and a step size of 2, curve increases at 2 units/sec

After 10 sec, the output has increased by 20 to 40