Solve the problem below on this paper in the spaces provided. In your solution you need to show not only the answers but the steps or rationale you used to arrive at the answer. If you perform special actions on your calculator (like a SOLVE or a cross product), write out the steps you used and precisely what you entered into the calculator. Your answers need to be complete enough to make your work checkable. Box your final answers. If you need more space, you may attach a paper with the continued part of the problem clearly designated as the continued part.

1. The North Koreans launch a rocket vertically upward with a constant acceleration, \(a_0\). (Who else would launch a rocket so that it could come down and blow up the launch pad when it comes back to earth?) The rocket burns fuel for a known time, \(\Delta t\), after which it proceeds upward in free flight. The rocket is tracked by an antenna, as shown, that is situated a known distance, \(d\), from the launch site. The antenna must rotate counter-clockwise to continue to follow the rocket. Assume that the height the rocket reaches is not high enough for \(g\) to change enough to care about—i.e., assume \(g\) is constant.

In answering the questions posed, take as known quantities \(a_0\), \(\Delta t\), \(g\), and \(d\). Give all answers in terms of these known variables.

a) After the fuel burn, what is the acceleration of the rocket? Explain.

\[
\text{Once the fuel burns, the rocket is in free flight and } a = -g \text{ (downward).} \checkmark
\]

b) What is the maximum velocity attained by the rocket and when does it occur?

\[
V_{\text{max}} = a_0 \Delta t @ t = \Delta t \checkmark
\]

c) What is the height of the rocket when the fuel is exhausted?

\[
y = \frac{1}{2} a_0 \Delta t^2 + V_0 \Delta t + y_0 = \frac{1}{2} a_0 \Delta t^2 = y_{fa} \checkmark
\]
d) What is the greatest height attained by the rocket?

\[
y_{\text{top}} = -\frac{1}{2} g t^2 + V_{\text{max}} t + y_{\text{fb}}
\]
\[
V_{\text{top}} = -gt + V_{\text{max}} = 0
\]
\[
t = \frac{V_{\text{max}}}{g} = \frac{a_0 dt}{g}.
\]
\[
y_{\text{top}} = \frac{a_0^2 dt^2}{2g} + \frac{a_0 dt}{g}
\]
\[
y_{\text{top}} = \frac{a_0^2 dt^2}{2g} + \frac{a_0 dt}{g}
\]
\[
y_{\text{top}} = \frac{a_0^2 dt^2}{2g} \left( \frac{a_0}{g} + 1 \right)
\]
\[
y_{\text{top}} = \frac{a_0^2 dt^2}{2g} \left( \frac{a_0}{g} + 1 \right)
\]

e) At what time does the rocket reach the apex of its trajectory?

\[
t_{\text{top}} = \Delta t + \frac{a_0 dt}{g} = \frac{\Delta t}{2} \left( 1 + \frac{a_0}{g} \right) V = t_{\text{top}}
\]

f) What is the total flight time of the rocket from launch until it impacts the earth again?

Need to add the drop time, \( t_{\text{down}} \).

\[
y = \frac{1}{2} a_0 t^2 + V_0 t + y_0 = -\frac{1}{2} g t_{\text{down}}^2 + \frac{a_0 dt^2}{2} \left( \frac{a_0}{g} + 1 \right) = 0
\]
\[
t_{\text{down}} = \sqrt{\frac{a_0 dt^2}{2} \left( \frac{a_0}{g} + 1 \right)} \quad t_{\text{tot}} = \Delta t \left( 1 + \frac{a_0}{g} \right) + \sqrt{\frac{a_0 dt^2}{2} \left( \frac{a_0}{g} + 1 \right)}
\]

g) At what height is \( v_r = 2v_\theta \)? Draw a vector diagram showing \( \vec{v}, \vec{v}_r, \) and \( \vec{v}_\theta \) in this situation.

\[\theta = \frac{h}{d} = \frac{v_r}{v_\theta} = 2, \quad h = 2d \]
\[\hat{e}_r \text{ not defined - straight path}\]

h) At what height is \( a_r = 2a_\theta \)? Draw a vector diagram showing \( \vec{a}, \vec{a}_r, \) and \( \vec{a}_\theta \) in this situation.

\[\text{H's a very similar diagram} \]

\[\hbar = 2d \]
i) Answer g) for the rocket’s downward trajectory. Draw the vector diagram also.

\[ \frac{\Delta h}{d} = \frac{v_r}{v_t} = 2 \]

\[ h = 2d \]

j) Answer h) for the rocket’s downward trajectory. Draw the vector diagram for this case too.

\[ h = 2d \]

k. What is the impact speed of the rocket at the launch site?

\[ v_{imp} = -g t + v_0, \quad t = t_{down} \]

\[ v = -g \sqrt{\frac{a_0}{g}} At^2 (\frac{a_0}{g} + 1) \]

\[ v = -\sqrt{a_0 g} \frac{At^2 (\frac{a_0}{g} + 1)}{2} \]

l) Out to the side of your vector diagram in g), show the unit vectors \( \hat{e}_n \) and \( \hat{e}_t \).

m) Do the same for the diagram in i).

n) At what height is \( v_r \) maximum?

\[ v_{r,\text{max}} = v_{\text{max}} = \frac{1}{2} a_0 At^2 \text{ from c).} \]

o) At what height is \( a_n \) minimum?

\[ a_n = 0 \text{ everywhere - straight path} \]

p) Develop an expression for \( \theta \) as a function of time for the initial flight phase before the fuel is exhausted.

See attached

q) Also would be interesting to draw a \( V vs t \) diagram.
n. $v_{\text{max}}$ when fuel runs out, @ $\frac{1}{2} a_0 t$
from $c$.

0. $a_n = 0$ everywhere; straight path

P. On way up before fuel burn-up,

\[ v = a_0 t \]
\[ y = \frac{1}{2} a_0 t^2 \]

\[ r = \sqrt{y^2 + d^2} = \sqrt{\frac{1}{4} a_0^2 t^4 + d^2} \]

\[ v_\theta = v \cos \theta = a_0 t \cos \theta \]

\[ v_\theta = r \dot{\theta} \]

\[ \dot{\theta} = \frac{v_\theta}{r} = \frac{a_0 t \cos \theta}{\sqrt{\frac{1}{4} a_0^2 t^4 + d^2}} \]