ME 422 – Quiz 2
Winter 2016

In giving your answer, the answer alone is not enough. Make sure you clearly give your rationale for arriving at the answer. It must be clear to me how you arrive at your answer. Points:

1. A type-1 system has a second open-loop pole at \( s = -6 \).

   a. Draw the root locus for this system.

   ![Root Locus Diagram]

   If the natural frequency of the closed-loop system is 5 rad/sec,

   b. Draw the locations of the closed-loop poles on your root locus above. Clearly mark their coordinates.

   c. What is the frequency of response for a step input into this system?

   \[ \omega_d = 4 \text{ rad/sec} \]

   d. What is the closed-loop system’s damping ratio (\( \zeta \))?

   \[ \cos \theta = \frac{3}{5} = 0.6 \]

   e. What is the percent overshoot of this step response?

   \[ %OS = e^{-5\pi/\sqrt{1-0.6^2}} = 0.0948 = 9.48\% \]

   f. What is the steady-state error for this step response?

   Type 1, so \( e_{ss} = 0 \) for step

35 pts = 50%
2. Answer the following questions related to the system above.

a. What is the closed-loop transfer function for this system?

\[
G_{CL} = \frac{NG}{DG + NG} = \frac{1}{s^3 + (1+2)s^2 + (a+2)s + a + 1}
\]

b. What is the characteristic equation for the system?

\[
s^3 + 3s^2 + (a+2)s + a + 1 = 0
\]

c. Analyze the stability characteristics of the system and give the range of values of the variable \(a\) for which the system is stable.

\[
a + 2 > 0 \Rightarrow a > -2
\]
\[
a + 1 > 0 \Rightarrow a > -1
\]

Sufficient condition:
\[
a_0 \cdot a_3 - a_1 \cdot a_2 < 0
\]

\[
(a+1) \cdot 1 - (a+2) \cdot 3 < 0
\]
\[
a + 1 - 3a - 6 < 0
\]
\[
-2a - 5 < 0
\]

Most restrictive condition is \(a > -1\)

\[
-2a < 5
\]
\[
a > -\frac{5}{2}
\]

\[
12
\]

\[
8
\]

\[
\text{ KP-\text{eu}} = \lim_{s \to 0} G_{OL} = \frac{1}{a}
\]

\[
30\text{pts}
\]
\[
\text{eu} = \frac{R_o}{1 + \frac{1}{a}} = \frac{R_o a}{a + 1}
\]

\[
2\text{of}2
\]