ME 422 – Quiz 4
Winter 2014

In giving your answer, the answer alone is not enough. Make sure you clearly give your rationale for arriving at the answer. It must be clear to me how you arrive at your answer.

Point values: a=2, b=3, c=3, d=5, e=3, f=3, g=4, h=3, i=5, j=4, k=6 = 4

<table>
<thead>
<tr>
<th>Input</th>
<th>Type 0</th>
<th>Type 1</th>
<th>Type 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$e_{ss}$</td>
<td>$e_{ss}$ constant</td>
<td>$e_{ss}$</td>
</tr>
<tr>
<td>Step</td>
<td>$\frac{R_0}{1 + K_{p-ess}}$</td>
<td>$K_{p-ess} = constant$</td>
<td>$\frac{R_0}{1 + K_{p-ess}}$</td>
</tr>
<tr>
<td>Ramp</td>
<td>$\frac{R_0}{K_v}$</td>
<td>$K_v = 0$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Parabola</td>
<td>$\frac{R_0}{K_a}$</td>
<td>$K_a = 0$</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

1. A system is type 1.
   a) What does this mean about the structure of its open-loop and/or closed-loop transfer function?

   2. The open-loop transfer function had an integrator (\(\frac{1}{s}\)) in it.

   b) How can you tell a type 1 system from its log mag plot alone?

   3. On the low frequency (left-hand) side, it starts off with a slope of -20 dB/dec.

   c) How can you tell a type 1 system from its phase plot alone?

   3. On the left-hand side, it starts off with a phase angle of -90°.
If the system's log mag plot, at its low-frequency end (with only one or two single components of the transfer function asserting themselves), passes through -30 dB at 0.01 rad/sec

d) What is the system's gain? Use log mag plot below to help with your thinking.

![Log Magnitude (dB) plot](image)

Curve depressed by \(20 \log k = -70 \text{ dB}\)

\[ k = 10^{-70/20} = 0.000316 \]

e) What is the system's magnitude (not log mag) at \(\omega = 0.001 \text{ rad/sec}\)?

\[ @ \quad \omega = 0.001, \quad 20 \log M = -10 \text{ dB} \]

\[ M = 10^{-10/20} = \frac{1}{10^0} = 0.316 \]
f) What would be the system's \( \phi \) at \( \omega = 0.001 \text{ rad/sec} \)?

At this frequency, only \( K \) & \( \frac{1}{S} \) are active. \( \phi_K = 0 \) & \( \phi_{\frac{1}{S}} = -90^\circ \), so

\[ \phi = -90^\circ \]

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3) The \( \phi \) at the gain crossover frequency (between 0.0001 & 0.001) is -90°

\[ \phi_M = 90^\circ \]

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4) In the log scale on which \( \omega \) is plotted, the half-decades occur at 0.00316, 0.0316, 0.316, etc. (rad/sec for units, and see log mag sheet for an example)

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h) What is \( \omega_{\Phi_M} \)? Explain.

\( \omega_{\Phi_M} \) is halfway between 0.0001 & 0.001 on the log scale for \( \omega \), so

\[ \omega_{\Phi_M} = 0.000316 \text{ rad/sec} \]

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i) If the system is given a unit ramp input of 1 ft/sec, what will be the steady-state error?

\( K_v \) is intersection of log mag curve with 0 dB. \( K_v = 0.000316 \text{ rad/sec} \)

\[ e_{ss} = \frac{R_o}{K_v} = \frac{1 \text{ ft/sec}}{0.000316 \text{ rad/sec}} \]

\[ e_{ss} = 3162 \text{ ft} \]
j) It is also observed that the system's asymptote on the log mag passes through -70 dB at $\omega = 10$ rad/sec. What third component has been encountered in moving toward higher frequencies on the log mag plot? Give its transfer function.

There is a 1st-order lead at $\omega = 1$ rad/sec.

$$G_{lead} = s + 1$$

k) A Bode test is run at $\omega = 0.00316$ rad/sec. The input signal is shown on the plot below. Draw the output signal that would result from the test on this plot. Give your calculations and reasoning.

\[ A + 0.00316 \]  
\[ 20 \log M = -20 \text{dB} \]

\[ M = 10^{-1} = \frac{1}{10} = 0.1 \]

So output amplitude is $\frac{1}{10}$ of input amplitude and $\phi = -90^\circ$. 