**B1. Electric Field I**

**GOAL**
- To determine the electric vector field for a point charge.
- To examine the spatial dependence of the strength of the electric field for a point charge.
- To determine how vectors are added to get the electric field for more than one point charge
- To understand the electric field for discrete and continuous charge distributions.

**EQUIPMENT**
- Computer with Charges and Fields simulation software.

**THEORY**
It is straightforward to do qualitative charge and force experiments with tape, plastic, fur, glass, wool, etc., like you did previously. However, for technical reasons it is quite difficult to do quantitative experiments in a simple lab setting measuring small forces and charges. Instead we are going to use a simulation program to explore the characteristics of the electric field, which will help us visualize and understand the concept of electric field.

**1a. Single point charge: electric vector field.**

Open the simulation Charges and Fields. It will look like Figure 1. You can position both positive (+) and negative (-) point "charges" and then examine the electric field by using a "test charge." Various properties are set using the "control box."

- The potential meter is not needed for this lab, so move it off to the side.
- Move a positive particle (charge = 1.0 nC) near the center of the screen. [Click on the charge, drag it, then release it.]
- Move a test charge (E-field sensor) to a spot near the positive charge. (You should now see the electric field vector at that location in space.)
- Grab the test charge and move it around paying attention to the magnitude and direction of the electric field (as shown in the figure below). Go closer/farther, above/below, left/right. If you want a view of the whole electric vector field at once, place many test charges at different locations.

Q1. By looking at the electric field vector what do you know about the sign of the test charge? (Answer questions in the space provided at the end of part I).

- On the sketches page below, make a sketch of the electric
vector field for the positive point charge. Be sure the relative magnitudes of your vectors are correct. Indicate the magnitude of the electric field at different locations by the length of the vector.

- Compare your sketch of the electric vector field with the simulation by turning on "Show E-field." NOTE: This simulation uses arrows of the same length (different from what you just did); the relative strength of the electric field is indicated by the intensity of the color, darker is stronger.

1b. Single point charge: electric field strength.
Now look at the spatial dependence of the strength of the electric field. NOTE: 1 N/C = 1 V/m.
- Check the "grid" box in the control box.
- Check the "show numbers" box in the control box.
- Move the positive charge to a grid intersection near the left side of the screen.
- Move the test charge (E-field sensor) away from the positive charge along a line. At several distances (0.2, 0.3, ..., 1.5 m,...) from the positive charge record the corresponding strength of the electric field in Table 1. Use the distance scale in the lower left corner or use the "tape measure."
- Plot the data on the coordinate axes provided below.

Q2. From your previous mathematical experience, list three different functions that have roughly this shape.

The electric field strength of a point particle is \[ |\vec{E}| = \frac{KQ}{r^2}, \] (Eq. 1)
where \( K \approx 8.99 \times 10^9 \text{N} \cdot \text{m}^2/\text{C}^2 \) is a constant, \( Q \) is the charge of the point particle causing the field, and \( r \) is the distance from the charge to the field location. Is this consistent with your data?

- Using Eq. 1, on the coordinate axes overlay the curve corresponding to \( Q = 1 \text{nC} \). (Hint: Calculate \( E \) for several different values of \( r \), plot the points on the coordinate axes, and connect with a smooth curve).

Q3. Do the data points (from the simulation) agree with the theoretical curve? How good is the agreement? What could be possible sources of discrepancy?

2. Superposition
- Now put two positive particles (each charge = 1.0 nC) near the center of the screen, with a horizontal separation of 1 m.
- Move a test charge (E-field sensor) to a spot near the two positive charges. (You should see the electric field vector at that location in space.)
- Grab the test charge and move it around paying attention to the magnitude and direction of the electric field. Go closer/farther, above/below, left/right. If you want a view of the whole electric vector field at once place many test charges at different locations.
- In the space provided below, make a sketch of the electric vector field for the two positive point charges. Be sure the relative magnitudes are correct. Indicate the magnitude by the lengths of your vectors.
- Compare your sketch with the simulation by turning on "Show E-field."

Q4. It is said that the electric field follows the principle of superposition: “the net electric field is the vector sum of the electric
fields due to each charge. Pick a point in the plane (be specific) and show that the net electric field that you measured is the vector sum of the individual electric fields from each charge.

Q5. What is the difference between a vector sum and a scalar sum? Pick a point in the plane (be specific) where it is clear that the net electric field is the result of a vector sum and not a scalar sum.

Part I - Report
Answers to questions (Q1-5). Use back of the page or additional sheet of paper if necessary.
Sketch of the electric vector field for a single positive point charge.

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>Electric Field (N/C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>1.3</td>
<td></td>
</tr>
<tr>
<td>1.4</td>
<td></td>
</tr>
</tbody>
</table>

Electric field strength vs. distance for a point charge.
Sketch of the electric vector field for two positive point charges.

Part II. Electric Charge Distributions

We will consider two types of charge distributions. Discrete distributions consist of any number of point charges. Continuous distributions have charge distributed everywhere over an extended region. In the following sections we will consider examples of each type. For both types we will use the principle of superposition to find the total electric field.

3. Discrete charge distributions.

For each of the following configurations (3a, 3b, 3c):

- Assume that the charges are point-like.
- Calculate the electric field at the point indicated with a “?” (show your work). NOTE: There is no electric charge at the location indicated by the question mark.
- Verify your answer by using the simulation. Clearly specify your calculated answer, the answer obtained from the simulation, and whether any possible discrepancy is significant (explain).

3a. Two positive charges are separated by 3 m and the point of interest (?) is halfway between them.
3b. Three charges are arranged with the 1 nC charge 1 m away from each of the negative charges. The point of interest (?) is halfway between the charges -2 nC and 1 nC.

3c. The three charges form an equilateral triangle of side 3 m. The point of interest (?) is halfway between the two bottom charges.
4. Continuous charge distributions

An extended charged object can be considered as a collection of point charges. To find the total electric field we consider the field created by each of the many points and add them together using superposition. When adding these contributions it typically results in an integral.

The following conceptual steps will help you:

- Define a helpful coordinate system so you can specify where the charge distribution is located.
- Pick a generic chunk of charge, \(dq\). (Do not choose a special spot like the middle or end.)
- Identify the place in space where you want to know the electric field, say \(P\).
- Determine the electric field due to the chunk of charge at \(P\). If the electric field is not along a coordinate axis you will need two expressions, one for each component.
- The electric field due to a small piece of charge, \(dq\), is the field produced by a point like charge:
  \[
  E = \frac{KE(dq)}{r^2}.
  \]
  Where “\(dq\)” represents an infinitesimal amount of charge.

- The total field is a sum (i.e. integral) over all of the pieces. So typically you will have separate “\(x\)” and “\(y\)” components:
  \[
  E_x = \int \left( \frac{K}{r^2} \right)_x dq \quad E_y = \int \left( \frac{K}{r^2} \right)_y dq
  \]
- Identify the variable of integration, this should be related to your coordinate(s).
- Now express \(dq\), \(r\), and any angular functions in terms of the variable of integration.

4a. We want an expression for the electric field of a uniformly charged rod at a location along the rod’s axis away from the rod. Specifically, we want to find the electric field at \(P\) as a function of the distance, \(a\), away from the end of the rod. (See figure below.) The final answer can only depend on the total charge on the rod, \(Q\), the length of the rod, \(L\), and the distance from the end of the rod, \(a\), along with physical constants. The following steps will guide you through the process.

![Diagram](image)
8 - Electric Field I

- On the diagram indicate the position, $P$, where you want to calculate the field. $P$ is on the axis of the rod. Label the distance between your point $P$ and the nearest end of the rod as $a$.
- Now, choose a small piece of charge, $dq$, on the rod. What is the distance from the chunk to the point $P$? You must identify an **integration variable that is geometric (like a distance or an angle)** because the field depends on distance (a geometric quantity). Since this is a rod, the integration variable should be a distance of some sort. Label this distance on your diagram.
- Remember that $E$ is a vector and you’ll need a coordinate axis. For the electric field at $P$, is it necessary to break $E$ up into $x$ and $y$ components? Explain.
- Your final task is to find ‘$r$’, ‘$dq$’, to put into the integral for this problem. In other words, find $dq$ and $r$ in terms of the integration variable.
- What are the relevant limits of your integral?
- Finally, perform the integral.
- For your result, what do you get for $Q = 15$ nC, $L = 2$ m, and $a = 1$ m?
- Check your prediction with the simulation software. Before you can test your prediction, you need to know if you’ve built a proper “uniformly charged rod”. How will you build a charged rod to test your answer? Where will you place the charges? How can you test that the charge distribution is symmetric? (Hint: If you click at a point that is a distance say $x = 1$ m away, how would that vector compare to the vector at the same point on the other side of the rod?)
- Does the simulated value match your prediction? Quantify any discrepancy and comment on the result. What are the possible sources of discrepancy?

4b. Repeat the procedure above to find the electric field at the center of semi-circular ring. Use a radius of 1 m and a total charge of 15 nC. Please note that you will have to consider the $x$ and $y$ components of the electric field separately. For this problem the most natural variable of integration is the angle $\theta$. 

Rev. Sep2013