The Adoption of New Technologies: Location, Learning and Asset Pricing Implications

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This paper analyzes the optimal investment strategy of two firms confronted with the option to adopt a new technology. I add two key features: location and learning. A firm gains relative advantage entirely due to its geographic placement—this is the location benefit. Firms also learn from the adoption experience of their rival—this is the learning benefit. Imperfect competition induces firms to adopt early while learning induces firms to wait. This tradeoff has two implications: First, firms in better locations should never adopt after their rivals; second, technology adoption should be geographically clustered. These implications are consistent with the direct evidence regarding technology adoption. Since investment in a new technology is a growth option, location and learning also affect asset prices. I show that firms’ risk loadings ($\beta$s) and returns correlate positively for geographically close firms.

Keywords: Geography, clustering, investment, imperfect competition, technology adoption

JEL classification: L11, L22, G11, G12, G31, R10, R11

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One of the most striking features of the geography of economic activity is its concentration—production is remarkably concentrated in space (Krugman (1991)). There is a small but burgeoning literature showing that technology adoption is also geographically concentrated (Kelley and Helper (1999), Baptista (2000) and No (2008)). Firms and workers are much more productive in large and dense urban environments than other locations. Both Smith (1904) and Marshall (1961) recognized the value of such dense urban environments that allow workers to interact more frequently. Workers learn from each other, and in the process of exchanging ideas, they effectively transfer knowledge regarding new technology. In this paper, I investigate the implications of location and learning on technology adoption by analyzing the geographical patterns of technology adoption and asset prices.

In the model, two firms have the option to adopt a new technology that reduces marginal cost of production. There are three key ingredients: competition, location and learning. First, consider the effect of competition. Adoption increases a firm’s flow of earnings, but adopting sooner is more expensive than adopting later. Under imperfect competition, since the earnings of a firm depend on its rivals, both firms engage in a value-destroying strategy of early adoption. Next, consider the effect of location. A firm’s location of production gives it a relative advantage. For example, one firm may be closer to an urban cluster and hence have a higher access to a pool of skilled workers. Traditionally, such location benefits are motivated with the label “transportation cost”. I assume that firm 1—which is in a better location—has a lower transportation cost than firm 2. This is the only source of asymmetry between firms.

Finally, consider the effect of learning. The cost of new technology has two components. The first component is the actual physical cost of technology. The second component is the soft information associated with the effective operation of the new technology. By definition, it is difficult to document this soft information and hence it can only be effectively transferred through interpersonal interactions.1 The importance of the transfer of such soft information should not be

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1Note that this information is not the same as the imperfect information as in Grenadier (1999).
underestimated. In a study of the adoption of machinery equipment technology, Teece (1977) estimates the cost of such transfers to be on average 20% of the total project costs. Learning decreases adoption cost.

Frequent interpersonal interactions are plausible when workers are embedded in a network that may be due to either geographical or social ties. It is natural to assume that the frequency of interactions between firms’ workers is high when the firms are near each other. Similarly, the frequency of interactions is high when the workers share common ties; for example, if they are alumni of the same university. Lastly, note that learning has a temporal dimension. Workers learn from prior adoption experience of their rival, which can only take place if firms sequentially invest; learning is impossible if both firms simultaneously invest.

Competition, location and learning cause the following trade-off: On one hand, early investment in the new technology by a firm leads to higher short term earnings. A portion of the increase in earnings of the leader, i.e., the adopting firm, comes at the expense of the follower. Due to its better location, firm 1 has a higher market share and hence is more likely to be the leader because it has a greater ability to appropriate the benefits from adoption. On the other hand, since the follower firm learns from the adoption experience of the leader, it benefits from the lower adoption cost. Learning increases the likelihood that the follower will adopt in the future, which in turn reduces the present value of the leader’s earnings.

The tradeoff leads to two pure strategy equilibria: one featuring sequential investment (SEQ) and the other featuring simultaneous investment (SIM). First, when firms are far from each other, investment in the new technology is sequential. Firm 1 invests first and after a significant lag, firm 2 follows suit. Observationally, technology adoption is geographically dispersed. Technology trickles down from firms located in better locations to others less well situated. Second, when firms are near each other, both firms simultaneous invest in the new technology. Observationally, technology adoption is geographically clustered; the lag in adoption timing is zero.

The intuition behind “it is easier to collude among equals” captures the essence of the paper.
First, consider the effect of competition and location while ignoring learning. When firms are near each other, the market share of firm 1 is not too different from firm 2. Therefore, firms tacitly collude and they decide to simultaneously invest. When firms are geographically far from each other, the market share of firm 1 is much higher than firm 2. Colloquially, firm 1 is a “Maverick”—it is unwilling to participate in any collusive action. Now add learning. Since firm 2 learns from firm 1’s adoption experience, firm 1 is more prone to collude and simultaneously invest.\(^2\)

Learning plays both a direct and an indirect role. In SEQ equilibrium, firm 2 learns from the adoption experience of firm 1—this is the direct role. When firms are geographically close, frequency of interactions between workers is high and hence learning benefits are high. But when firms are geographically close, both firms simultaneously invest and in this outcome there is no learning. This seemingly counterintuitive result arises because learning also serves as a threat—this is the indirect role. Firm 1 is afraid that it will not be the sole leader for long when learning benefits are high and hence it will not adopt early.

The decision to adopt a new technology is ultimately a growth option possessed by both firms. Competition, location and learning affect the investment strategy of both firms and therefore they also have asset pricing implications. First, consider the case in which firms are geographically distant and when firm 1 has already adopted. Firm 2 has the growth option to adopt the new technology. Any good news in the product market has an asymmetric effect on both firms. Positive news increases the probability that firm 2 will adopt and hence it decreases the future earnings of firm 1. Therefore, competition acts like a natural hedge. Second, consider the case in which firms are geographically near each other. Both firms simultaneously invest in the new technology—the strike price of the option is the same. Therefore, any positive news affects both firms symmetrically. This leads to two testable implications that highlight the impact of geography on asset pricing. When firms are geographically close, both firm risk ($\beta$) and stock returns correlate positively.

To summarize, the model is consistent with a variety of stylized facts regarding technology

\(^2\)I use the terminology of “Maverick” from Ivaldi, Jullien, Rey, Seabright, and Tirole (2003) who present a report concerning the economics of tacit collusion to the European Commission.
adoption and asset pricing. First, learning from interactions positively affects the probability of technology adoption, consistent with the evidence summarized in Young (2009). Second, the model predicts positive correlation between size and the probability of technology adoption, consistent with Karshenas and Stoneman (1993), Stoneman and Kwon (1996), Baptista (2000), and Hall and Khan (2003). Third, the model predicts geographical clustering of technology adoption, consistent with Kelley and Helper (1999), Baptista (2000), and No (2008). Fourth, the model relies on the fact that adopting first is more beneficial than adopting second, which is consistent with Karshenas and Stoneman (1993), Stoneman and Kwon (1996), and Baptista (2000). Fifth, the model generates positive correlation of stock returns among geographically close firms, consistent with Pirinsky and Wang (2006), Eckel, Lffler, Maurer, and Schmidt (2011), Barker and Loughran (2007), and Wongchoti and Wu (2008). Finally, the model predicts that stock returns co-move together in less concentrated industries, consistent with Hoberg and Phillips (2010), and Bustamante (2011).

The paper is organized as follows: Section I summarizes previous work on technology adoption and its asset pricing implications. Section II introduces a simple two-period deterministic model, which provides the basic intuition of the paper. Section III derives equilibrium and considers its properties. Section IV summarizes major empirical implications of the model and relates them to existing evidence. In Section V, I generalize the two period setup by introducing an infinite period stochastic model, which allows me to derive asset pricing implications. Section VI concludes the paper.

I. Related Literature

Ever since the pioneering studies of hybrid corn adoption by Ryan and Gross (1943) and Griliches (1957), the empirical literature in economics has emphasized the impact of learning from interpersonal interactions on technology adoption. Ryan and Gross (1943) found that the most important determinant of adoption of hybrid seeds by Iowa farmers is their interactions with neighbors. The importance of learning from interactions has since been corroborated in various studies
of technology adoption across industries and across countries. Young (2009) provides a survey of
the empirical findings.

The importance of these interactions has also been documented in an international setting.
Economic activities such as trade and foreign direct investment are positively correlated with tech-
Comin and Hobijn (2010) and Comin, Dmitriev, and Rossi-Hansberg (2012) find that technology
adoption trickles down geographically. Firms located in countries close to adoption leaders adopt
first and then the technology slowly trickles down to firms located in countries farther away.

The intuition behind the effect of learning from interactions is straightforward, but in isolation
it ignores competition. In a study of the adoption of computerized numerically controlled machines,
Stoneman and Kwon (1996) finds that profits of non-adopters decrease as their rivals adopt. There
is overwhelming evidence that firm size and the probability of adoption are positively correlated
(Stoneman (2002)). Grenadier (1996) illustrates how imperfect competition can lead to boom
and bust cycles in real-estate. The seminal paper that incorporates the effect of competition and
technology adoption is Fudenberg and Tirole (1985) (hereafter referred to as FT). In their model,
FT develop the adoption decision of two firms in an infinite period deterministic setup. In a
continuous time setting, it is not obvious what it means when a rival instantaneously reacts to the
competitor’s actions. FT formally solve this issue and derive a closed-loop equilibrium—a sub-game
perfect equilibrium where firms do not pre-commit to their strategies.

Carlson, Dockner, Fisher, and Giammarino (2011) and Bustamante (2011) extend FT’s model
by deriving asset pricing implications.\(^4\) They both clarify the role of imperfect competition on
asset prices. For example, Bustamante (2011) (along with Hoberg and Phillips (2010)) find that
stock returns in less concentrated industries co-move together. Bena and Garlappi (2010) find an

\(^3\)Caselli and Coleman (2001) documents positive correlation between trade openness and computer adoption across
countries

\(^4\)Numerous studies develop the link between asset pricing and real options. Starting with the seminal article Berk,
Green, and Naik (1999), Gomes, Kogan, and Zhang (2003), Kogan (2004), Zhang (2005), Cooper (2006), Aguerrevere
(2009), and Novy-Marx (2011) link real options, product market competition, and asset pricing. These papers assume
pure competition or monopoly competition, or singular investment costs, which are slightly different than my setting.
analogous result in a similar setup that links technological innovation and asset prices.

I add location and learning to FT’s framework. This allows me to incorporate the evidence regarding both learning and competition on technology adoption. Additionally, I also highlight how geography impacts asset pricing.

II. A deterministic model of technology adoption

This section develops a deterministic two-period model to analyze the adoption decision of each of the two firms. I augment FT’s framework by adding the effect of location and learning from interpersonal interactions. I highlight the positive effect of market size or market share on technology adoption by explicitly considering product market competition.

A. The setup

A.1. Exogenous demand in the product market

Consider an environment with two periods, \( t \in \{0, T\} \), where \( T \) is the length of time spanned by the two periods. Firms 1 and 2 are rivals in the homogenous product market where they produce quantities \( q_1 \) and \( q_2 \) in each period \( t \).

For tractability, I assume that the demand curve is linear. Denoting the market clearing price in period \( t \) by \( P \), the inverse demand function is

\[
P(X_t, Q_t) = X + X_t - Q_t,
\]

where \( Q_t \equiv q_{1t} + q_{2t} \) is the industry output in period \( t \) and parameters \( X_t \geq 0 \) along with \( X \) cause parallel shifts in the demand level. Parameter \( X \), whose lower bound is given below, ensures that the demand level is sufficiently high so that both firms produce positive quantities. The demand level in the second period, \( X_T \), is proportional to the first period demand level \( X_0 \), so that \( X_T = X_0 e^{\mu T} \), where \( \mu \) is the demand growth rate.
Lastly, management of both firms discount the cash flows in period $T$ by the discount factor $e^{-rT}$. I further assume a transversality like condition: $r > \mu$.

A.2. Variable costs

Variable costs are composed of two components: manufacturing cost denoted by $m$ and transportation cost denoted by $l_i$. Transportation cost arises due to firm’s location. I assume that transportation cost of firm 1, which is in a better location, is lower than that of firm 2, so that $l_1 < l_2$. Differences in location is the only asymmetry between the two firms.

Initially, both firms use the old technology; they have the same manufacturing costs $m$. Manufacturing cost of the adopting firm decreases to $m - \alpha$ for $\alpha > 0$. No further technological advances are anticipated.\(^5\)

A.3. Effect of learning on the cost of adoption

In order to understand the effect of learning from interpersonal interactions, a distinction must be made in the two components comprising adoption costs.\(^6\)

The first component of the adoption cost concerns the physical equipment itself. For example, consider the adoption of computer numerically controlled (CNC) machines.\(^7\) A CNC machine does not need to be controlled by an operator; it can be programmed to be run by a computer, thereby increasing productivity. The first component is the actual physical cost of CNC machine.

The second component concerns soft information needed to effectively operate the technology. This information consists of the methods of organization and operation, quality control, and other manufacturing procedures. By definition, this soft information is difficult to codify—this information is transferred effectively only through interpersonal interactions. In the example of CNC

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\(^5\)Total variable cost $m + l_i$ is pivotal. Alternatively, one can assume that firms differ in their variable cost due to exogenous reasons. I attribute the difference in the variable cost to the difference in transportation cost to match the empirical evidence.

\(^6\)This subsection relies heavily on the note in Arrow (1969) and the empirical evidence in Teece (1977).

\(^7\)Some studies that consider the adoption of CNC are Kelley and Helper (1999), Baptista (2000), and Stoneman (2002).
machines, the second component of the adoption cost is related to the implementation of effective management practices.

Finally, note that learning from interactions has a temporal dimension. Learning is only possible in case of sequential investment as the non-adopting firm learns from the adoption experience of their rival. Learning reduces adoption cost in the second period. Learning is impossible when both firms simultaneously invest. Table I summarizes the adoption costs in the two situations:

<table>
<thead>
<tr>
<th>Situation</th>
<th>Investment</th>
<th>Adoption costs of firm i</th>
<th>Adoption costs of firm j</th>
</tr>
</thead>
<tbody>
<tr>
<td>If both firms adopt in the either period</td>
<td>Simultaneous</td>
<td>No</td>
<td>0</td>
</tr>
<tr>
<td>If firm i adopts in the first period and firm j adopts in the second period</td>
<td>Sequential</td>
<td>Yes</td>
<td>0</td>
</tr>
</tbody>
</table>

Table I: Effect of learning on adoption cost

$\kappa \in (0,1)$ represents learning benefits. Learning lowers the second period adoption cost by a factor $1 - \kappa$. For example, if the firms are geographically close to each other, then $\kappa \gg 0$, which implies that firm $j$—which may adopt after firm $i$—learns from the adoption experience of firm $i$.

A.4. Timing and state variables that determine Cournot quantities in each period

Figure 1 shows the timing of the game in both periods. Each period involves two stages: the first stage concerns the adoption decision, and the second stage concerns production. Each firm observes the demand level $X_t$ at the beginning of each period. Subsequently, each firm decides whether or not to adopt the new technology. Prior to production, each firm also observes its rival’s adoption decision. Afterward, both firms produce quantities $q_{it}$. Formally, there are three state variables: demand level $X_t$, a discrete variable $\theta_i \in \{0,1\}$, which takes a value of 1 if firm $i$ adopts
(and 0 otherwise) and another discrete variable \( \theta_j \in \{0,1\} \), which takes a value of 1 if firm \( j \) adopts (and 0 otherwise).

![Figure 1: Timing of the game](image)

Mathematically, there is mapping from state variables \( (X_t, \theta_i, \theta_j) \) to firm \( i \)'s quantity \( q_i \) and earnings \( \pi_i \):

\[
(X_t, \theta_i, \theta_j) \mapsto q_i(X_t, \theta_i, \theta_j) \text{ and } \pi_i(X_t, \theta_i, \theta_j).
\]

The following assumption regarding \( X \) ensures positive supply by each firm:

**ASSUMPTION 1.** \( X > 2l_2 - l_1 + m > 0 \).

### B. Effect of Competition

#### B.1. Quantities and earnings per period

Assumption of linear inverse demand and constant marginal cost leads to closed form expressions for equilibrium quantities and earnings in each period. Cournot quantities are linear in demand level \( X_t \):

\[
q_i(X_t, \theta_i, \theta_j) = \frac{1}{3} \left[ X + VC_j(\theta_j) - 2VC_i(\theta_i) \right] + \frac{X_t}{3} \quad \text{for } i, j \in \{1,2\},
\]  

(1)
where
\[ VC_i(0) = m + l_i \text{ and } VC_i(1) = m + l_i - \alpha \quad \text{for } i \in \{1, 2\} \]
is the total variable cost of firm \( i \). Earnings are a quadratic function of the demand level \( X_t \):
\[ \pi_i(X_t, \theta_i, \theta_j) = e_{i0}(\theta_i, \theta_j) + e_{i1}(\theta_i, \theta_j)X_t + e_{i2}(\theta_i, \theta_j)X_t^2, \tag{2} \]
where
\[ e_{i0}(\theta_i, \theta_j) = \frac{e(\theta_i, \theta_j)^2}{9}; \quad e_{i1}(\theta_i, \theta_j) = \frac{e(\theta_i, \theta_j)}{9}; \quad e_{i2}(\theta_i, \theta_j) = \frac{1}{9}; \quad \text{and } e(\theta_i, \theta_j) \equiv X + VC_j(\theta_j) - 2VC_i(\theta_i). \]

Upon inspection, quantity \( q_i \) and earnings \( \pi_i \), depend on the relative transportation cost between firms \( VC_i(\theta_i) - VC_j(\theta_j) \). The difference leads to a series of inequalities given in the following Lemma.

**LEMMA 1.** Adoption by firm \( i \) negatively affects firm \( j \):

(i) **Quantity and earnings of firm \( i \) decreases as firm \( j \) adopts:**
\[ k(X_t, 1, 0) > k(X_t, 1, 1) > k(X_t, 0, 0) > k(X_t, 0, 1) \text{ for } k \in \{q_i, \pi_i\}. \]

(ii) **Supply increases when either firm adopts:**
\[ q_i(X_t, 1, 1) + q_j(X_t, 1, 1) > q_i(X_t, 1, 0) + q_j(X_t, 0, 1) > q_i(X_t, 0, 0) + q_j(X_t, 0, 0). \]

(iii) **Increase in earnings is higher for firm \( i \) when it adopts first:**
\[ \pi_i(X_t, 1, 0) - \pi_i(X_t, 0, 0) > \pi_i(X_t, 1, 1) - \pi_i(X_t, 0, 1). \]

Suppose firm 1 adopts in the first period. Firm 1’s quantity produced increases from \( q_1(X_0, 0, 0) \) to \( q_1(X_0, 1, 0) \). In response, firm 2’s quantity decreases from \( q_2(X_0, 0, 0) \) to \( q_2(X_0, 0, 1) \). In aggregate, the total supply increases from \( q_1(X_0, 0, 0) + q_2(X_0, 0, 0) \) to \( q_1(X_0, 1, 0) + q_2(X_0, 0, 1) \) and
hence market clearing price decreases. Firm 1’s earnings increase from $\pi_1(X_0, 0, 0)$ to $\pi_1(X_0, 1, 0)$ and firm 2’s earnings decreases from $\pi_2(X_0, 0, 0)$ to $\pi_2(X_0, 0, 1)$. This means that some of the gain in earnings for firm 1 after adoption come at the expense of firm 2.

The feature that adoption has a negative effect on the rival is due to the assumption of Cournot competition. This result is consistent with Stoneman and Kwon (1996) who find that earnings of non-adopters decrease as the number of adopters increase. Generally, this result will arise as long as the goods produced by both firms are strategic substitutes.

Consider the effect of competition and location without learning. The last inequality implies that adoption gains depend on the order of adoption: Adopting first is better than adopting second. This leads both firms to adopt in the first period. Now, add learning. Consider the limiting case where the learning benefits are drastic so that $\kappa \approx 1$. Suppose firm $i$ adopts in the first period. Firm $j$ adopts in the second period for sure since adoption cost is zero due to drastic learning. Consequently, firm $i$ does not remain the sole user of the new technology, which in turn reduces its incentive to adopt in the first period. Herein lies the fundamental tradeoff: competition induces firms to adopt early while learning induces firms to wait.

In the next section, I formally analyze the tradeoff by deriving equilibrium.

## III. Equilibrium derivation

### A. Equilibrium Overview

There are two interesting possibilities concerning the relative timing of investment in new technology. First, the relative timing can be sequential in nature in which firm $i$ invests in the first period and becomes the leader. Firm $j$, which is naturally the follower, has the option to invest in the second period. Second, the relative timing can be simultaneous in nature in which both firms simultaneously invest in either period.

Of course, there is a third possibility: Neither firm invests in the new technology in either period.
Intuitively, both firms will never invest in new technology if the demand level in the first period is sufficiently low: $X_0 \leq X_0$. The expression for $X_0$ is provided later in this section. Heuristically, the threshold $X_0$ is the demand level at which it is just feasible for firm 1 to invest in the first period. This no-investment possibility is not interesting and so from here onwards in this deterministic setup, I assume that $X_0 > X_0$.

The strategy of firm $i$ involves adoption time denoted by $\tau_i$ and quantity $q_i$ produced in both periods. Each firm must contemplate its value if it is the leader or the follower, or if it simultaneously invests in either period. Equilibrium strategies depend on the present value of earnings in each possibility. The equilibrium concept is that of sub-game perfect equilibrium, which rules out empty threats. In case of multiple equilibria that can be Pareto ranked, I assume that firms coordinate on the Pareto-Superior equilibrium.

Formally, a sub-game perfect equilibrium is a pair of strategies $[\tau^*_{i}, \{q^*_i\}_{t\in\{0,T\}}]$ for $i \in \{1, 2\}$ such that each firm maximizes the present value of earnings for every demand level $X_t$ given the equilibrium strategy of the rival.

B. Firm value as a follower

Suppose firm $j$ has already invested in the new technology in the first period so that it becomes the leader. Firm $i$ as a follower has the option to adopt in the second period.

Let $V^F_i(X_0)$ denote the value of the follower firm $i$. The follower’s value consists of two components: The first component represents the present value of earnings from not adopting the new technology. This is the value from assets in place. The second component represents the growth option that denotes the increase in earnings when firm $i$ adopts the new technology. Mathematically,

$$V^F_i(X_0) = W^F_i(X_0) + O^F_i(X_0),$$

(3)
where

\[ W_F^i(X_0) = \pi_i(X_0, 0, 1) + \pi_i(X_T, 0, 1) e^{-rT} \]  

is the value from assets in place and

\[ O_F^i(X_0) = \max \left( \pi_i(X_T, 1, 1) - \pi_i(X_T, 0, 1), 0 \right) e^{-rT} \]
\[ = A_i^F e^{-rT} \max(X_T - X_i^F, 0); \text{ and } X_i^F \equiv \frac{B_i^F + (1 - \kappa)I_0}{A_i^F} > 0. \]

is the growth option that represents the increase in earnings when firm \( i \) adopts with

\[ A_i^F = \frac{4\alpha}{9} > 0; \text{ and } B_i^F = \frac{4}{9} \alpha(2l_i - l_j + m - X) < 0. \]

Investment thresholds \( X_i^F \) of both firms are:

\[ X_2^F = 2l_2 - l_1 + m - \frac{9I_0(1 - \kappa)}{4\alpha}; \text{ and } X_1^F = X_2^F - 3(l_2 - l_1). \]

Upon inspection of equation (5), follower firm \( i \) is long \( A_i^F \) number of calls with a strike price of \( X_i^F \). Also note that the follower threshold for firm 1 (\( X_1^F \)) is lower than the follower threshold for firm 2 (\( X_2^F \)).

The following technical assumption regarding the adoption cost in the second period ensures that the investment threshold for firm 1 as a follower is positive.

**ASSUMPTION 2.** Adoption cost \( I_0 \) is sufficiently high: \( I_0 \geq \frac{4\alpha(l_2 - 2l_1 - m + X)}{9(1 - \kappa)} \).

Comparative statics of firm 2’s investment threshold is given in the following lemma.\(^8\)

\(^8\)I focus on comparative statics of follower firm 2’s investment threshold as firm 1 will never be the follower in equilibrium. The qualitative features of the comparative statics also hold for \( X_1^F \).
LEMMA 2. The investment threshold of follower firm 2, $X^F_2$

(i) increases if relative transportation cost $(l_2 - l_1)$ increases,

(ii) increases if efficiency of the new technology $(\alpha)$ increases,

(iii) decreases if learning benefits $(\kappa)$ increases.

As the relative transportation cost $l_2 - l_1$ increases, the quantity produced by firm 2 decreases. Therefore, it has less incentive to adopt a variable cost reducing technology since the adoption costs are sunk ($(1 - \kappa)I_0 \gg 0$). Similarly, an increase in the efficiency of new technology $\alpha$ increases quantity produced by firm 1. In response, quantity produced by firm 2 decreases, which again lowers the incentive of firm 2 to adopt in the second period. Finally, the adoption cost decreases for firm 2 since it learns from the adoption experience of firm 1.

The level of firm 2’s investment threshold is further understood by comparing it with the monopoly threshold as shown below.

B.1. Effect of learning and market structure

Suppose firm 2 is a monopolist. Standard calculations yield that per period earnings with and without adoption are

$$\pi^M_2(X_t, 1) = \frac{1}{4} (X_t + X - m - l_2 + \alpha)^2 \quad \text{and} \quad \pi^M_2(X_t, 0) = \frac{1}{4} (X_t + X - m - l_2)^2$$

respectively. Similar to the analysis above, firm 2 adopts if earnings from adoption exceed earnings from non-adoption. Intuitively, firm 2 adopts if the demand level $X_T$ in the second period is sufficiently high. This condition is equivalent to $X_T > X^M_2$ where

$$X^M_2 \equiv \frac{-\alpha}{2} + l_2 + m + \frac{2I_0}{\alpha} - X.$$ (7)
Comparing equations (6) and (7) yields

\[ X_2^F - X_2^M = \frac{\alpha}{2} + l_2 - l_1 + \frac{I_0(9(1 - \kappa) - 8)}{4\alpha}. \]  

Figure 2 plots \( X_2^F - X_2^M \) as a function of the learning benefits \( \kappa \) for low and high values of relative transportation cost \( l_2 - l_1 \). Increases in learning decreases \( X_2^F - X_2^M \) while increases in relative transportation cost increases \( X_2^F - X_2^M \).

When firms are geographically far from each other so that (i) learning benefits are low \((\kappa \approx 0)\) and (ii) relative transportation costs are high \((\text{high } l_2 - l_1)\), firm 1 effectively raises the adoption cost of firm 2 by investing in the first period. This result is similar in spirit to Salop and Scheffman (1983), who indicate the use of predatory pricing in raising rivals’ costs. Equation (8) points to an alternate mechanism: First period investment by firm 1 raises second period adoption cost for firm 2. On the other hand, when firms are geographically close, learning reduces the adoption cost of firm 2.

Proposition 1 summarizes the economic reasoning behind equation (8).

**PROPOSITION 1.** If firms are geographically far from each other, then the investment threshold of follower firm 2 in a duopoly is higher than the investment threshold of firm 2 in a monopoly.
This result reverses if firms are geographically close.

Firm 1 as a leader needs to evaluate the benefits of adopting in the first period taking the negative impact of learning into account. The next section, which precisely calculates incentives of both firms to become the leader, formalizes this notion.

C. Firm value as a leader

Let $V^L_i(X_0)$ denote the value of the leader firm $i$ after it adopts in the first period, conditional upon follower firm $j$ pursuing its optimal exercise strategy. The leader’s value consists of two components: The first component represents the present value of earnings assuming follower firm $j$ never adopts. This is the value from assets in place. The second component represents the loss in earnings in the second period when firm $j$ adopts. Mathematically,

$$V^L_i(X_0) = W^L_i(X_0) + O^L_i(X_0),$$

where

$$W^L_i(X_0) = \pi_i(X_0, 1, 0) + \pi_i(X_T, 1, 0) e^{-r_T} - I_0$$

is the value from assets in place and

$$O^L_i(X_0) = \left[ \begin{array}{c} 1(X_T \geq X^F_j) \\ \text{Indicator that reflects that firm } j \text{ adopts} \end{array} \right] \left[ \begin{array}{c} \pi_i(X_T, 1, 1) - \pi_i(X_T, 1, 0) \\ \text{Loss in earnings when firm } j \text{ adopts} \end{array} \right] e^{-r_T},$$

$$= A^L_i \left[ 1(X_T \geq X^F_j) (X_T - X^L_i) \right] e^{-r_T}; \quad X^L_i \equiv \frac{B^L_i}{A^L_i},$$

17
represents loss in earnings if firm \( j \) adopts with

\[
A_i^L = \frac{-2\alpha}{9} < 0; \text{ and } B_i^L = \frac{1}{9}\alpha(3\alpha - 4l_j + 2l_i - 2m + 2X) > 0.
\]

Firm \( i \) as a leader wants the demand level in the first period \( X_0 \) to increase\(^9\) but it does not want the demand to increase too much. Firm \( j \) adopts in the second period if the demand increases too much, which reduces the earnings of the leader firm \( i \). This is akin to firm \( i \) being short a call option. Specifically, firm \( i \) is short firm \( j \)'s growth option. However, the option is not a typical call option since the exercise event \( \{X_T > X_j^F\} \) is different from the exercise price \( X_i^L \).

Increase in learning benefits reduces the investment threshold of follower firm \( j \), which reduces \( O_i^L \). The following lemma summarizes the effect of learning on the value of being the leader.

**Lemma 3.** Learning decreases the present value of earnings from being the leader.

The next section describes how competition may lead to early investment.

**C.1. Effect of competition on early adoption**

There are two different cases. First, suppose the first period demand level \( X_0 = x \) where \( x \) is such that firm 2 prefers being the leader over being the follower, i.e.,

\[
V_2^L(x) - V_2^F(x) > 0.
\]

Since firm 1 has a lower transportation cost than firm 2, it has to be the true that firm 1 also prefers being the leader over being the follower, i.e.,

\[
V_1^L(x) - V_1^F(x) > 0.
\]

\(^9\)Recall that the demand level in the second period \( X_T = X_0e^{\mu T} \).
That is both firms find it beneficial to be leader. Therefore, firm 1 decides to invest at a slightly lower threshold $x - \epsilon$. In turn, firm 2 decides to invest at $x - 2\epsilon$. This process continues up to a threshold $X_p^{21}$, which is the point at which firm 2 is indifferent between being the leader or the follower. Formally, $X_p^{21}$ is the solution to

$$V_L^2(y) - V_F^2(y) = 0.$$  \hspace{1cm} (12)

In the appendix, I show that $X_p^{21} \in (0, X_F^2)$ and it is unique.

Note that at $X_p^{21}$, firm 1 is still better off being the leader or being the follower, i.e.,

$$V_L^1(X_p^{21}) - V_F^1(X_p^{21}) > 0.$$ 

Now consider the second case where both firms act myopically. They ignore their rival’s response. Each firm optimally invests at $X_i^{LN} \equiv X_j^F e^{-\mu T} - \epsilon$. Each firm invests at a point where it knows that their rival as a follower does not invest in the second period. Since $X_1^F < X_2^F$, $X_2^{LN} < X_1^{LN}$.

Finally consider the best responses that combine these two cases. In the event $\{X_1^{LN} < X_p^{21}\}$, then firm 2 does not want to be the leader anyway, and firm 1 invests at $X_0 = X_1^{LN}$ without any fear of being preempted by firm 2. In the complementary event $\{X_1^{LN} \geq X_p^{21}\}$, firm 2 wants to be the leader. But from the argument above, due to competition, the investment threshold decreases to $X_p^{21}$.

To summarize, firm 2 will never be the leader. Firm 1 invests in the first period at a threshold

$$X_0 \geq \min\{X_1^{LN}, X_p^{21}\}.$$ 

Lastly, each firm compares its sequential investment possibility with the simultaneous investment possibility, which is shown next.
D. Firm value with simultaneous investment

D.1. Simultaneous investment in the second period

Let \( V_S^i(X_0) \) denote the value of firm \( i \) simultaneously investing with firm \( j \) in the second period. The simultaneous investment value consists of two components: The first represents the present value of earnings when neither firm invests in the new technology. This is the value from assets in place. The second component represents the growth option, which denotes the increase in earnings when both firms simultaneously invest in the new technology. Mathematically,

\[
V_S^i(X_0) = W_S^i(X_0) + O_S^i(X_0),
\]

where

\[
W_S^i(X_0) = \frac{\pi_i(X_0, 0, 0)}{e^{-rT}} + \frac{\pi_i(X_T, 0, 0)}{e^{-rT}},
\]

is the value from assets in place and

\[
O_S^i(X_0) = \max \left( \frac{\pi_i(X_T, 1, 1) - \pi_i(X_T, 0, 0)}{e^{-rT}}, 0 \right) - \frac{I_T}{e^{-rT}} \]

is the growth option that represents the increase in earnings when firm \( i \) simultaneous adopts with

\[
A_i^S = \frac{2\alpha}{9} > 0; \quad B_i^S = \frac{-1}{9} \alpha (\alpha - 4l_i + 2l_j - 2m + 2X) < 0.
\]
The investment threshold of firm $i$ simultaneously investing with firm $j$, $X_i^S$, is

$$X_i^S = \frac{-\alpha}{2} - l_1 + 2l_2 + m + \frac{9l_0}{2\alpha} - X$$

and

$$X_1^S = X_2^S - 3(l_2 - l_1). \quad (16)$$

Upon inspection of equation (15), firm $i$ is long $A_i^S$ number of calls with a strike price of $X_i^S$.

Comparing the follower investment threshold for both firms in equation (6) with the simultaneous investment threshold for both firms in equation (16) yields a series of inequalities as given in the following lemma.

**LEMMA 4.** The follower and simultaneous investment threshold are ranked as follows:

$$0 < X_1^F < X_2^F < X_1^S < X_2^S$$

Due to the higher transportation cost of firm 2, i.e., $l_2 > l_1$, it is intuitive that $X_1^F < X_2^F$ and $X_1^S < X_2^S$. The reason why $X_2^F < X_1^S$ is subtle. Market clearing price decreases as any firm adopts (lemma 1). Thus, when both firms simultaneously adopt, there is a significant drop in price. In response, both firms simultaneously invest when the demand level in the second period is really high.

In the case of simultaneous investment, both firms have to coordinate investing at either firm 1’s investment threshold $X_1^S$ or firm 2’s investment threshold $X_2^S$. The following lemma shows that firm 2 simultaneously invests with firm 1 at firm 1’s investment threshold $X_1^S$.

**LEMMA 5.** Firm 2 prefers simultaneously investing with firm 1 over being the follower, i.e.,

$$V_2^S(X_0) > V_2^F(X_0) \quad \forall X_0 \geq 0.$$  

**Proof.** Three different cases arise:

**Case 1:** $X_T < X_2^F$ — In this case, the growth options are worthless: $O_2^F = O_2^S = 0$. From Lemma 1, it is clear that $W_2^S > W_2^F$.

**Case 2:** $X_2^F < X_T < X_1^S$ — In this case, follower firm 2’s growth option is positive and the simultaneous growth option is worthless: $O_2^F > 0$ and $O_2^S = 0$. Follower firm 2’s value $V_2^F$ increases
with the demand level $X_0$. Then the difference $V_2^S(X_0) - V_2^F(X_0)$ is at a minimum when learning benefits are drastic $\kappa \approx 1$ and when $X_0 = X_1^S e^{-\mu T}$:

$$V_2^S(X_1^S e^{-\mu T}) - V_2^F(X_1^S e^{-\mu T}) \bigg|_{\kappa=1} = -\frac{(e^{\mu T} - e^{r T})(4\alpha^2 + 12\alpha(l_2 - l_1)) + 4\alpha(e^{\mu T} - e^{r T})}{\alpha} > 0,$$

where $X = 2l_2 - l_1 + m + \alpha + a$ with $a > 0$ (Assumption 1).

**Case 3:** $X_T > X_1^S$ — In this case, both growth options are positive in value: $O_2^F = O_2^S > 0$.

From Case 2 and $O_2^S > 0$, it has to be that $V_2^S(X_0) > V_2^F(X_0)$.

**COROLLARY 1.** Both firms simultaneously invest in the second period in the event $\{X_T > X_1^S\}$.

**Proof.** This is a direct implication of Lemmas 4 and 5.

If firm 1 decides to invest simultaneously, it will invest in the new technology in the second period if $X_T > X_1^S$. Firm 1 will invest with the understanding that firm 2 follows suit. This understanding is implicit—there is no contractual obligation that enforces firm 2 to simultaneously invest. Therefore firm 2 may deviate and invest in the first period. The implication of Lemma 5 is that firm 2 does not credibly deviate. If firm 2 deviates and invests in the first period, then firm 1 invests at a lower threshold in the first period, which makes firm 2 the follower and firm 2 does not want to be the follower.

**D.2. Simultaneous investment in the first period**

Intuitively, both firms adopt in the first period if the demand level is really high. This is equivalent to the condition that

$$\pi_2(X_0, 1, 1) + e^{-rT} \pi_2(X_T, 1, 1) - I_0 > V_2^S(X_0).$$

Firm 2’s value from simultaneous investment in the first period
Both firms simultaneously adopt in the first period in the event \( \{ X_0 > \overline{X}_0 \} \) where \( \overline{X}_0 \) is the solution to

\[
\pi_2(y, 1, 1) + e^{-rT} \pi_2(y e^{\mu T}, 1, 1) - I_0 - V_2^S(y) = 0. \]

Armed with investment thresholds and firm values in every single possibility, I derive equilibrium next.

### E. Equilibrium selection

To summarize the optimal strategies, neither firm invests in the new technology if the demand level in the first period is sufficiently low, i.e., \( X_0 < \underline{X}_0 \). Also, both firms invest simultaneously in the first period if the demand level is sufficiently high, i.e., \( X_0 > \overline{X}_0 \). In the intermediate case, \( X_0 \in (\underline{X}_0, \overline{X}_0) \), two possibilities arise: First, firm 1 invests in the first period and firm 2 invests in the second period in the event \( \{ X_T > X_2^F \} \). Second, both firms simultaneously invest in the second period in the event \( \{ X_T > X_1^S \} \).

Figure 3 displays the normal form game with only dominated strategies when \( X_0 \in (\underline{X}_0, \overline{X}_0) \). The boxed values denote sub-optimal best responses. For example, \( \boxed{V_2} \) in the top right panel denotes sub-optimal present value of earnings of firm 2. If firm 1 invests in the first period, the best response of firm 2 is to invest in the second period in the event \( \{ X_T > X_2^F \} \) (not \( \{ X_T > X_1^S \} \)). Similarly, in the bottom left panel, the best response of firm 1 if firm 2 invests in the event \( \{ X_T > X_2^F \} \) is to invest in the first period. Sub-optimal firm value \( \boxed{V_1} \) represents firm 1’s present value of earnings if it invests in the second period in the event \( \{ X_T > X_1^S \} \).

I distinguish two cases:

- **Case A:** \( V_1^L(X_0) > V_1^S(X_0) \);
- **Case B:** \( V_1^L(X_0) \leq V_1^S(X_0) \).

First, consider Case A. The top left panel featuring sequential investment is the unique equilibrium. In this case, firm 1 invests first in the first period and firm 2 retains the option to invest
in the second period.

Next, consider Case B. There are two pure strategy equilibria: Sequential investment in the top left panel and simultaneous investment in the bottom right panel. Any convex combination of these two equilibria is also an equilibrium — these combinations form the set of mixed strategy equilibria.

From Lemma 5, $V_2^S > V_2^F$ and since $V_1^S > V_1^L$, both firms are better off simultaneously investing with each other. Formally, SIM equilibrium Pareto dominates SEQ and all other mixed strategy equilibria. The next lemma summarizes this result.

**Lemma 6.** *Simultaneous investment equilibrium, if it exists, Pareto dominates (from the firm’s point of view) all other equilibria.*

In order to reduce the set of equilibria in Case B, I make the following assumption:

**Assumption 3.** *(Equilibrium Selection)* *Both firms simultaneously invest if $V_1^L(X_0) < V_1^S(X_0)$.*

Next proposition summarizes the investment strategies.
PROPOSITION 2. The investment strategies of both firms are as follows:

(i) If $X_0 \leq X_0^*$, then neither firm invests in the new technology in either period,

(ii) If $X_0 \geq X_0^*$, then both firms invests in the new technology in the first period,

(iii) If $X_0 \in (X_0, X_0^*)$ and $V_{1L}(X_0) > V_{1S}(X_0)$, then firm 1 invests in the first period and firm 2 invests in the second period in the event $\{X_T > X_2^F\}$,

(iv) If $X_0 \in (X_0, X_0^*)$ and $V_{1L}(X_0) \leq V_{1S}(X_0)$, then both firms invest in the second period in the event $\{X_T > X_1^S\}$.

Comparative statics are given in the following lemma.

LEMMA 7. The incentive of firm 1 to invest in the first period increases if

(i) learning benefits ($\kappa$) decrease: $\frac{\partial(V_{1L} - V_{1S})}{\partial \kappa} < 0$,

(ii) relative transportation cost ($l_2 - l_1$) increases: $\frac{\partial(V_{1L} - V_{1S})}{\partial(l_2 - l_1)} > 0$,

(iii) efficiency of new technology ($\alpha$) increases: $\frac{\partial(V_{1L} - V_{1S})}{\partial \alpha} > 0$.

When firms are near each other, relative transportation cost $l_2 - l_1$ is low and learning benefits $\kappa$ are high. Thus, firm 1’s incentive to be the leader is low. On the other hand, learning does not impact firm value from simultaneous investment. Therefore, geographically close firms find it mutually beneficial to simultaneously invest. Observationally, technology adoption is geographically clustered and lag in adoption timing between the two firms is low.

The following two corollaries elaborate the combined effect of learning and competition.

COROLLARY 2. With learning, competition may not cause firms to invest early.

A well-known notion in the real options literature is that competition erodes the time value of
waiting.\textsuperscript{10} Corollary 2 places caveats on this notion. With simultaneous investment, both firms actually invest at a higher threshold \( X^S_1 \). Observationally, neither firm adopts for a long time, but when they adopt, they both adopt together.

**COROLLARY 3.** *If learning benefits are high (when firms are close to each other), then firms simultaneously invest which features no learning*

Learning serves a dual role. In sequential investment, firm 2 learns from the adoption experience of firm 1. This is the learning in the traditional sense. However, learning reduces the value of the leader. The threat of learning induces firm 1 to simultaneously invest and as a result, neither firm learns. Figure 4 shows this counterintuitive result graphically.

### IV. Empirical Implications

Now, I summarize the empirical implications of the model, relating them to existing evidence.

\textsuperscript{10} Grenadier (2002) and Back and Paulsen (2009) show that the investment threshold approaches the positive NPV threshold as the intensity of competition increases. They assume singular investment costs but do not place restrictions on capacity increases from the new option. On the other hand, I place a restriction on the capacity increase due to new technology (only one growth option) but relax the assumption of singular investment costs. The fact that competition does not erode the option value to wait in a competitive setting has also been shown in Fudenberg and Tirole (1985), Pawlina and Kort (2006), Novy-Marx (2007), Carlson et al. (2011), and Bustamante (2011).
(i) **Positive effect of learning from interactions on technology adoption:** The magnitude of learning from interactions leads to two different equilibria: First high learning benefits lead to simultaneous investment—the lag in adoption timing between firms is zero. Second, low learning benefits lead to sequential investment—learning decreases the follower’s threshold. Therefore, unequivocally, learning reduces lag in adoption timing. Young (2009) provides positive evidence of social learning in technology adoption. Caselli and Coleman (2001), Keller (2002), Comin and Hobijn (2004), and Comin and Hobijn (2010) find that trade and foreign direct investment (activities which increase contact with foreign persons) increase the probability of technology adoption.

(ii) **Positive effect of size on technology adoption:** The model predicts a positive correlation between size and probability of technology adoption, which is corroborated in Karshenas and Stoneman (1993), Stoneman and Kwon (1996), and Baptista (2000). This idea goes back at least to Schumpeter [see the account in Hall and Khan (2003)]. He argued that larger firms adopt early since the benefits are greater for them. In the model, the variable cost of firm 1 is lower than that of firm 2. Therefore, it produces more and hence the benefits of adopting a variable cost reducing technology is higher for firm 1.

(iii) **Geographic clustering of technology adoption:** The model predicts simultaneous investment when firms are geographically close to each other. This is corroborated in Kelley and Helper (1999), Baptista (2000), and No (2008).

(iv) **Effect of location on technology adoption:** In an international study of technology adoption across various industries, Keller (2002), Comin and Hobijn (2004), Comin and Hobijn (2010), Comin et al. (2012) find significant regional asymmetries. Firms located in countries that are closer to “adoption leaders” invest earlier in the new technology. Technology slowly trickles down to firms less well situated.

(v) **Effect of adopting first:** The increase in earnings from adopting first is higher than the
increase in earnings from adopting second. This is corroborated in Karshenas and Stoneman (1993), Stoneman and Kwon (1996), and Baptista (2000).

These implications provide direct evidence of technology adoption. Since, location and learning affect investment strategies in the new technology, they also affect asset prices. This is next.

V. A stochastic model of technology adoption

Since investment in the new technology is a growth option held by both firms, location and learning also affect asset prices. In this section, I highlight how location and learning affect the risk profiles of firms.

A. Environment

Demand level shocks follow geometric Brownian motion\(^\text{11}\):

\[
\frac{dX_t}{X_t} = \mu dt + \sigma dW_t. \tag{17}
\]

Furthermore, there is a unique stochastic discount factor whose dynamics are

\[
\frac{dM_t}{M_t} = -rdt - \lambda dW_t, \tag{18}
\]

where \(\lambda \equiv \frac{g-r}{\sigma}\) is the market price of risk.

A.1. Game set up

The dynamic game is a repeated version of the game in Figure 1. After observing \(X_t\) at time \(t\), both of firms decide whether to adopt or not adopt. This decision is public knowledge—both firms know

\(^{11}\text{Uncertainty is modeled in the following way. The probability space is denoted by } (\Omega, \mathcal{F}, \mathbb{P}) \text{ where the filtration } (\mathcal{F}_t)_{t \geq 0} \text{ represents the information resolved over time and is known to both the firms.}\)
their rival’s adoption status also. Afterward, firms receive Cournot earnings, which is followed by realization of \( X_{t+dt} \) and the game is repeated.

I formalize the game below. Denote the first time when firm \( i \) adopts by \( \tau_i \) and define

\[
D_i(t) = \begin{cases} 
1 & \text{if } t \geq \tau_i \\
0 & \text{if } t < \tau_i 
\end{cases}
\]

The function \( D_i(t) \) is the Heaviside step function which takes a value of 1 when firm \( i \) adopts. At each instant, both firms make their investment decision knowing the complete history of the game \( \Phi_t = ([X_s]_{s \leq t}, [D_1(s), D_2(s)]_{s \leq t}) \), which is common to both the players. Given the Markov nature of the environment, it is natural to restrict attention to the most recent state variables \( X_t \) and \( D(t^-) \equiv [D_1(t^-), D_2(t^-)] \) where \( D_i(t^-) \equiv \lim_{s \uparrow t} D_i(s) \).

The firm \( i \) value is

\[
M_t V_i(X_t) = \mathbb{E}_t \left[ \int_t^\infty M_s \times \pi_i(X_s, \theta_i, \theta_j) \times 1[D_i(s) = \theta_i, D_j(s) = \theta_j] \, ds \right] \\
- \mathbb{E}_t \left[ M_{\tau_i} \times I_0 \times ((1-\kappa)1[D_i(\tau_i) = 0, D_j(\tau_i) = 1] + 1[D_i(\tau_i) = 1, D_j(\tau_i) = 1]) \right]
\]  

The first term in the integral is the present value of the flow of earnings and the second term is the present value cost under either sequential or simultaneous investment. I further assume that

\[
\delta > \frac{r + \sigma^2}{2};
\]

where \( \delta \equiv g - \mu \) to ensure finite firm value.

A pure strategy Markov-perfect equilibrium is a pair of strategies \( (\tau^*_i, \{q^*_i\}_{s \geq t}) \), for \( i \in \{1, 2\} \) such that each firm maximizes their firm value in equation (19) for every state \( [X_t, D(t^-)] \) given the equilibrium strategy of the rival.

\[\text{[12]Since } X_t \text{ is known, } M_t \text{ is known also.}\]
Similar to the previous section, I use backward induction to derive the equilibrium of the game.

B. Firm Value

B.1. Firm value as a follower

Assuming firm $j$ has already adopted, present value of firm $i$ at time $t$ is

$$M_t V_i^F(X_t) = \sup_{\tau_F^i \in T} \mathbb{E}_t \left[ \int_t^{\tau_F^i} \pi_i(X_s, 0, 1) M_s \, ds + \int_{\tau_F^i}^{\infty} \pi_i(X_s, 1, 1) M_s \, ds - M_{t_F^i}(1 - \kappa)I_0 \right]$$

$$= W_i^F + O_i^F. \quad (20)$$

where $\tau_F^i = \min\{t : X_t \geq X_F^i\}$ and $T$ is the set of stopping times. In the appendix, I show that

$$W_i^F(X_t) = \frac{e_{i0}(0, 1)}{r} + \frac{X_t e_{i1}(0, 1)}{\delta} + \frac{X_t^2 e_{i2}(0, 1)}{2\delta - r - \sigma^2}$$

is the value from assets in place and

$$O_i^F(X_t) = \begin{cases} \left( \frac{X_t}{X_F^i} \right)^{\gamma} \left( \frac{A_i^F X_F^i}{\delta} - \frac{B_i^F}{r} - (1 - \kappa)I_0 \right), & \text{if } X_t \leq X_F^i \\ \frac{A_i^F X_t}{\delta} - \frac{B_i^F}{r} - (1 - \kappa)I_0, & \text{if } X_t > X_F^i \end{cases}$$

is the value of the growth option where

$$X_F^i = \frac{B_i^F}{r} + \frac{(1 - \kappa)I_0}{\frac{A_i^F}{\delta}} \frac{\gamma}{\gamma - 1}; \quad \gamma = \frac{1}{\sigma^2} \left[ -(r - \delta - \frac{1}{2} \sigma^2) + \sqrt{(r - \delta - \frac{1}{2} \sigma^2)^2 + 2r\sigma^2} \right] > 1.$$
B.2. Firm value as a leader

Present value of firm $i$ upon immediate investment at time $t = \tau^L_i$ is

$$M_t V^L_i(X_t) = \mathbb{E}_t \left[ \int_{t}^{\tau^F_i} \pi_i(X_s, 1, 0) M_s \, ds + \int_{\tau^F_i}^{\infty} \pi_i(X_s, 1, 1) M_s \, ds \right] - I_0$$

$$= W^L_i + O^L_i. \quad (21)$$

In the appendix, I show that

$$W^L_i(X_t) = \frac{e_{i0}(1, 0)}{r} + \frac{X_t e_{i1}(1, 0)}{\delta} + \frac{X_t^2 e_{i2}(1, 0)}{2\delta - r - \sigma^2} - I_0$$

is the value from assets in place and

$$O^L_i(X_t) = \begin{cases} \left( \frac{X_t}{X^F_j} \right)^{\gamma} \left( \frac{A^L_i X^F_j}{\delta} - \frac{B^F_i}{r} \right), & \text{if } X_t \leq X^F_j \\ \frac{A^L_i X_t}{\delta} - \frac{B^L_i}{r}, & \text{if } X_t > X^F_j \end{cases}$$

is the loss in earnings when firm $j$ adopts.

It is also useful to consider optimal investment strategy of firm $i$ when it knows that it will be the leader. This is the hypothetical value when firm $j$ credibly pre-commits to be the follower. Formally,

$$M_t V^{LN}_i(X_t) = \sup_{\tau^L_i \in T} \mathbb{E}_t \left[ \int_{t}^{\tau^L_i} \pi_i(X_s, 0, 0) M_s \, ds + \int_{\tau^L_i}^{\tau^F_j} \pi_i(X_s, 1, 0) M_s \, ds + \int_{\tau^F_j}^{\infty} \pi_i(X_s, 1, 1) M_s \, ds \right]$$

$$- \mathbb{E}_t \left[ M^{LN}_i I_0 \right]$$

$$= W^{LN}_i + O^{LN}_i. \quad (22)$$
where $\tau^L_{i} = \min \{ t : X_t \geq X^L_{i} \}$. In the appendix, I show that

$$W^L_{i}(X_t) = \frac{e^{i0}(0,0)}{r} + \frac{X_t e^{i1}(0,0)}{\delta} + \frac{X^2_t e^{i2}(0,0)}{2\delta - r - \sigma^2}$$

is the value from assets in place and

$$O^L_{i}(X_t) = \begin{cases} \left( \frac{X_t}{X^L_{i}} \right)^{\gamma} V^L_i(X_t), & \text{if } X_t \leq X^L_{i} \\ V^L_i(X_t), & \text{if } X_t > X^L_{i} \end{cases}$$

is the value of the growth option where

$$X^L_{i} = \frac{B^L_{i} + I_0}{A^L_{i}}; \quad A^L_{i} = A^S_{i} - A^L_{i}; \quad B^L_{i} = B^S_{i} - B^L_{i}.$$

### B.3. Firm value with simultaneous investment

The present value of firm $i$ when it simultaneously invests with firm $j$ is

$$M_i V^S_i(X_t) = \sup_{\tau^S_{i} \in T} \mathbb{E}_{t} \left[ \int_{t}^{\tau^S_{i}} \pi_i(X_s, 0, 0) M_s ds + \int_{\tau^S_{i}}^{\infty} \pi_i(X_s, 1, 1) M_s ds - M_{\tau^S_{i}} I_0 \right]$$

$$= W^S_i + O^S_i. \quad (23)$$

where $\tau^S_{i} = \min \{ t : X_t \geq X^S_{i} \}$. In the appendix, I show that

$$W^S_i(X_t) = \frac{e^{i0}(0,0)}{r} + \frac{X_t e^{i1}(0,0)}{\delta} + \frac{X^2_t e^{i2}(0,0)}{2\delta - r - \sigma^2}$$

is the value from assets in place and

$$O^S_i(X_t) = \begin{cases} \left( \frac{X_t}{X^S_{i}} \right)^{\gamma} \left( \frac{A^S_i X^F_i}{\delta} - \frac{B^S_i}{r} - I_0 \right), & \text{if } X_t \leq X^S_{i} \\ \frac{A^S_i X_t}{\delta} - \frac{B^S_i}{r} - I_0, & \text{if } X_t > X^S_{i} \end{cases}$$
is the value of the growth option from simultaneous investment where

\[ X_i^S = \frac{B_i^S}{A_i^S} + I_0 \frac{\gamma}{\gamma - 1}. \]

C. Equilibrium properties

For simplicity, assume that the demand level \( X_t \) is low so that neither firm wants to invest immediately. The qualitative properties of the equilibrium remain the same as in Proposition 2.

First consider the properties in sequential investment. Without any fear of preemption, firm 1 optimally invests at \( \tau_t^{LN} \). On the other hand, if \( V_2^L(X_t^{LN}) - V_2^F(X_t^{LN}) > 0 \), then firm 2 finds it beneficial to be the leader over being the follower. Therefore, firm 2 preempts and the investment threshold decreases to \( X_{21}^{p1} \) (equation (12)) at which point, firm 2 is indifferent between being the leader or the follower. Therefore, firm 1 invests at the first time demand level reaches \( \min\{X_t^{LN}, X_{21}^{p1}\} \).

Second consider the properties in simultaneous investment. Since transportation cost of firm 1 is lower than that of firm 2, the only candidate for a simultaneous investment threshold is \( X_1^S \). For simultaneous investment to occur, Firm 1’s value as a leader has to be lower than firm 1’s value from simultaneous investment, i.e.,

\[ V_1^S(x) > V_1^L(x) \quad \forall x \in (0, X_1^S). \]

Otherwise, firm 1 invests when the demand level reaches \( \min\{X_t^{LN}, X_{21}^{p1}\} \).

In the appendix, I show that when firms are geographically close, then both firms simultaneously invest. Observationally, there is not a significant lag between adoption timing of both firms. On the other hand, when firms are geographically distant, then investment in the new technology is sequential in nature. Firm 1—which is in a better location—invests at the first time demand level \( X_t^{LN} < X_{21}^{p1} \), firm 1 invests without fear of any preemption by firm 2. Carlson et al. (2011) label this equilibrium as Leader-Follower non-preemptive equilibrium. On the other hand, if \( X_t^{LN} > X_{21}^{p1} \), firm 1 fears preemption by firm 2. Carlson et al. (2011) label this equilibrium as Leader-Follower preemptive equilibrium.
reaches \( \min \{ X^{LN}_t, X^{21}_p \} \). Firm 2 invests at the first time demand level reaches \( X^F_p \). Observationally, there is a significant lag between adoption timing of both firms. The next proposition summarizes the equilibrium.

**PROPOSITION 3.** The Markov perfect equilibrium is characterized by

(i) **Simultaneous equilibrium:** If firms are sufficiently close to each other, i.e. \( l_2 - l_1 < l^* \), then both firms simultaneously adopt at \( \tau^S_1 \).

(ii) **Sequential equilibrium:** If firms are sufficiently distant from each other, i.e. \( l_2 - l_1 > l^{**} \), then firm 1 adopts at \( \tau^{LN}_1 \) and firm 2 adopts at \( \tau^F_2 \). There is no credible preemption attempt by firm 2. In the intermediate case, i.e. \( l^* \leq l_2 - l_1 \leq l^{**} \), firm 1 adopts at \( \tau^{21}_p \) where \( \tau^{21}_p = \min \{ t : X_t \geq X^{21}_p \} \). Firm 1 adopts earlier than optimal due to the fear of preemption by firm 2. Firm 2 adopts at \( \tau^F_2 \).

In the next section, I analyze how location and learning affect firm risk.

**D. Effect of geography on firms’ risk loadings (\( \beta_s \))**

Following Carlson, Fisher, and Giammarino (2004), the dynamic betas of each firm are

\[
\beta_i(X_t) = \frac{\partial V_i(X_t)}{\partial X_t} \frac{X_t}{V_i(X_t)}. \tag{24}
\]

Upon inspection, risk loading (\( \beta \)) of each firm depends on the germane equilibrium as given in Proposition 3, which in turn depends on the relative transportation cost \( l_2 - l_1 \). In this manner, geography affects asset pricing.

Left panel in Figure 5 plots the relationship between the firm \( \beta \) and the demand level \( X_t \) under simultaneous investment while the right panel plots the relationship under sequential investment. One result is clear: the risk dynamics are markedly different for different equilibria. Consider the risk loading of firm 1 in SEQ equilibrium after it has already adopted. Any positive news
Figure 5: The left panel plots firms’ $\beta$s from simultaneous investment. The right panel plots firms’ $\beta$s from sequential investment assuming firm 1 has already invested. Solid line is for firm 1 and dashed line is for firm 2. The parameters are $\alpha = 3.5$, $X = 10.1$, $m = 4$, $l_1 = 0.5$, $l_2 = 1$, $r = 0.1$, $\delta = 0.09$, $I_0 = 150$, $\kappa = 0.65$, and $\sigma = 0.2$.

(higher values of $X_t$) increases the probability that follower firm 2 adopts. Therefore, positive news in the product market is dampened by the competitors growth option—competition acts like a natural hedge. The effect of competition is enough so that the risk loadings of both firms correlate negatively. In the SIM equilibrium, positive news in the product market affects both firms in the same direction. This leads to the following proposition.

**PROPOSITION 4.** When firms are geographically close, their risk loadings correlate positively.

**Proof.** This follows directly from Proposition 3. When firms are geographically close, the relative transportation cost $l_2 - l_1 < l^*$ and hence both firms simultaneously invest. \qed

**E. Effect of geography on correlation between stock returns**

Since conditional CAPM holds, firm $i$’s expected return (from equation (19)) is

$$E_t[R_i(X_t)] - rdt = \beta_i(X_t)\lambda; \text{ where } R_i(X_t) \equiv \frac{\pi_i(X_t; \theta_i, \theta_j) dt + dV}{V(X_t)}.$$
Figure 6: Solid line plots correlation under sequential investment and dashed line plots correlation under simultaneous investment. The parameters are $\alpha = 3.5$, $X = 10.1$, $m = 4$, $l_1 = 0.5$, $l_2 = 1$, $r = 0.1$, $\delta = 0.09$, $I_0 = 150$, $\kappa = 0.65$, $\sigma_{e_1} = \sigma_{e_2} = 0.2$, and $\sigma = 0.2$.

Assuming that realized returns also have an idiosyncratic component\footnote{This is outside the model but incorporating the idiosyncratic piece is straightforward.} with variance $\sigma_{e_i}^2$, correlation between the returns of both firms can be computed. Figure 6 plots correlation of returns for different equilibria. As expected, the correlations are markedly different for different equilibria. Under sequential investment (solid line), the correlation actually becomes negative while under simultaneous investment (solid line), correlation remains positive. This leads to the following proposition.

**PROPOSITION 5.** When firms are geographically close, returns co-move together.

*Proof.* This follows directly from Proposition 3. When firms are geographically close, the relative transportation cost $l_2 - l_1 < l^*$ and hence both firms simultaneously invest.

\[\square\]

**F. Empirical implications of geography on asset pricing**

Now, I summarize the empirical implications that concern asset pricing.

(i) **Local co-movement of stock returns:** When firms are geographically close, they simultaneously invest. Therefore, their $\beta$s’ and correlation co-move together. Pirinsky and Wang

(ii) **Effect of competition on stock returns:** In concentrated industries, so that \( l_2 - l_1 \) is large, investment is sequential. The firms’ \( \beta \)s do not co-move together. On the other hand, in less concentrated industries, so that \( l_2 - l_1 \) is small, both firms simultaneous invest. Then \( \beta \)s of firms co-move together. This is corroborated in Hoberg and Phillips (2010) and Bustamante (2011) who find that stock returns co-move together in less concentrated industries.

(iii) **Effect of geography on investment:** When firms are geographically close, they simultaneously invest. Dougal, Parsons, and Titman (2012) find that corporate investment is indeed geographic clustered.

**VI. Conclusion**

This paper presents a model that explains the role of location and learning from interpersonal interactions on technology adoption. The setup involves two firms who have the option to adopt a new technology. While the logic of the positive role of learning is straightforward, the logic either explicitly or implicitly ignores imperfect competition. Imperfect competition introduces strategic effects. One one hand, competition induces firms to adopt early and on the other hand, learning induces firms to wait. There are two equilibria that arise from the tradeoff—these equilibria depend on the relative location of the firms. First, firms located in better locations never adopt after firms less well situated. Observationally, there is geographic dispersion in technology adoption. Second, technology adoption is geographically clustered. Observationally, the lag in adoption timing is small for geographically close firms. To summarize, the spatial and temporal patterns of technology adoption are two sides of the same coin. Technology adoption diffuses through time and also through space.

Since location and learning affect the investment decision, they also affect asset prices. Specif-
ically, the risk dynamics and returns of geographically close firms correlate positively. These predictions are testable, although one has to be careful since location choice by itself is endogenous. While the understanding of industrial organization on asset prices has been studied (Hoberg and Phillips (2010) and Bustamante (2011)), I am not aware of any studies that document the impact of geography on the cross section of stock returns and investment. Therefore, I anticipate future work testing the predictions of the model.
Appendix A. Proof that $X_P^{21} \in (0, X_2^F)$

Define

$$\xi_2(x) \equiv V_2^L(x) - V_2^F(x)$$

and note that $X_P^{21}$ is the value so that $\xi_2(X_P^{21}) = 0$. I show that $X_P^{21} \in (0, X_2^F)$ by intermediate value theorem. Upon inspection, it is clear that $\xi_2(0)$ is negative and $\xi_2(X_2^F)$ is positive. Therefore, it has to be that $\xi_2(x)$ crosses zero at least once.

Appendix B. Derivation of the Firm Value

Risk Neutral Measure

Define random variable $L_T$ by

$$L_T = \frac{dQ}{dP} \text{ on } \mathcal{F}_T,$$

where $Q$ is known as the risk-neutral measure. Since $Q << P$, i.e. $Q$ is absolutely continuous with $P$ on $\mathcal{F}_T$, we also have that $Q << P$ on $\mathcal{F}_t$ for all $t \leq T$. We define

$$L_t = \frac{dQ}{dP} \text{ on } \mathcal{F}_t \quad 0 \leq t \leq T.$$

That is, for every $t$ we have that $L_t \in \mathcal{F}_t$, so $L$ is an adapted process known as the likelihood process. Let the dynamics of $L_t$ be given by

$$dL_t = -L_t \lambda d\tilde{W}_t.$$

Then by standard calculations,

$$M_s = M_t e^{-r(s-t)} \frac{L_s}{L_t}.$$
Denote the price of any contingent claim, which is a function \( g(X_s) \) by \( V \). Then we have that

\[
M_t V(X_t) = \mathbb{E}_t \left[ \int_t^\infty M_s g(X_s) \, ds \right] = \mathbb{E}_t \left[ \int_t^\infty M_t e^{-r(s-t)} \frac{L_s}{L_t} g(X_s) \, ds \right]
\]

\[
V(X_t) = \mathbb{E}^Q_t \left[ \int_t^\infty e^{-r(s-t)} g(X_s) \, ds \right]
\]

The price of any contingent claim is simply the discounted cash flows (where discounting is done with the risk free rate) in the risk neutral measure. The dynamics of \( X_t \) in the risk neutral measure are

\[
dX_t = (r - \delta) dt + \sigma d\mathbb{W}_t.
\]

\[
\mathbb{E}^Q_t [X_s] = X_t e^{(r-\delta)(s-t)}; \quad \mathbb{E}^Q_t [X^2_s] = X_t^2 e^{2(r-\delta) + \sigma^2(s-t)}.
\] (B1)

**Firm value from assets in place**

The present value of flow of earnings from assets in place is

\[
W_i = \mathbb{E}^Q_t \int_t^\infty \pi_i(X_s, \theta_i, \theta_j) e^{-r(s-t)} \, ds
\]

\[
= \mathbb{E}^Q_t \left[ \int_t^\infty \left[ e_{i0}(\theta_i, \theta_j) + e_{i1}(\theta_i, \theta_j) X_s + e_{i2}(\theta_i, \theta_j) X^2_s \right] e^{-r(s-t)} \, ds \right]
\]

\[
= \int_t^\infty e_{i0}(\theta_i, \theta_j) e^{-r(s-t)} \, ds + \int_t^\infty e_{i1}(\theta_i, \theta_j) \mathbb{E}^Q_t [X_s] e^{-r(s-t)} \, ds + \int_t^\infty e_{i2}(\theta_i, \theta_j) \mathbb{E}^Q_t [X^2_s] e^{-r(s-t)} \, ds
\]

\[
= \frac{e_{i0}(\theta_i, \theta_j)}{r} + \frac{e_{i1}(\theta_i, \theta_j) X_t}{\delta} + \frac{e_{i2}(\theta_i, \theta_j) X^2_t}{2\delta - r - \sigma^2}.
\]

\( W^F_i, W^L_i \) and \( W^S_i \) can be calculated by substituting appropriate values of the \( \theta_i \) and \( \theta_j \).

**Derivation of follower growth option** \( O^F_i(X_t) \)

It is convenient to use the derive the following Lemma. Define \( \tau_M = \min\{t : X_t \geq M\} \). Then

**LEMMA 8.**

\[
\mathbb{E}_t [e^{-r(\tau_M - t)}] = \left( \frac{X_t}{M} \right)^\gamma
\]
where \( \theta \) is given in the paper.

**Proof.** Define the random process

\[
Y(s) = \int_t^s (r - \delta - 0.5\sigma^2) \, ds + \int_t^s \sigma d\mathbb{W}^Q.
\]

Upon inspection, it is clear that \( Y_s \) is normally distributed. Now, consider the process \( \{e^{-r(s-t)} e^{\gamma y(s)}\}_{s \geq t} \).

For \( t \leq \tau_M \), this process is a bounded martingale in the risk neutral measure if the coefficient \( \gamma \) is the positive solution to the equation

\[
e^{-r(s-t)} E[e^{\gamma Y(s)}] = 1.
\]

The negative solution makes the equation unbounded. The term in the expectation is simply the moment generating function of \( Y(s) \). Standard calculations yield the equation for \( \gamma \) as given in the paper.

By optional sampling theorem, we have that

\[
e^{-r(\tau_M-t)} E[e^{\gamma Y(\tau_M)}] = 1; \quad \text{or} \ E[e^{-r(\tau_M-t)} e^{\gamma \log(MX_t)}] = 1.
\]

This simplifies to

\[
e^{\gamma \log(M/X_t)} E[e^{-r(\tau_M-t)}] = 1.
\]

Slight algebra yields the quantity to be proved above.

**Note that the proof is quite general. Specifically, the proof and the methodology can be generalized to levy process that do not have any upward jumps.** For example, the proof can be easily extended to rare disasters.

With Lemma 8 at hand, consider the follower firm \( i \) value when it follows a trigger strategy.
That is, suppose that firm $i$ adopts when the demand level reaches a threshold $M$. That is

$$M = X_t e^{y(\tau_M)}.$$ 

The first value from this arbitrary strategy is

$$O^F_i(X_t, M) = \mathbb{E}^Q_t \left[ \left( \int_{\tau_M}^{\infty} (A^F_i X_s - B^F_i) e^{-r(s-t)} ds \right) - (1 - \kappa)I_0 e^{r(\tau_M-t)} \right]$$

$$= \mathbb{E}^Q_t \left[ e^{-r(\tau_M-t)} \left( \int_{\tau_M}^{\infty} (A^F_i X_s - B^F_i) e^{-r(s-\tau_M)} ds - (1 - \kappa)I_0 \right) \right]$$

$$= \mathbb{E}^Q_t \left[ e^{-r(\tau_M-t)} \mathbb{E}^Q_{\tau_M} \left[ \left( \int_{\tau_M}^{\infty} (A^F_i X_s - B^F_i) e^{-r(s-\tau_M)} ds - (1 - \kappa)I_0 \right) \right] \right]$$

$$= \mathbb{E}^Q_t \left[ e^{-r(\tau_M-t)} \left( A^F_i M \delta - B^F_i \right) - (1 - \kappa)I_0 \right]$$

$$= \left( \frac{X_t}{M} \right)^\gamma \left( A^F_i M \delta - B^F_i \right) - (1 - \kappa)I_0.$$

Now, I solve for optimal threshold $X^F_i$. This can be simply done by differentiating $O^F_i(X_t, M)$ with respect to $M$ and setting that equal to zero. Solving that equation yields the optimal threshold $X^F_i$. It is also easy to show that second order condition is negative when $X_t = X^F_i$.

**Derivation of simultaneous growth option $O^S_i(X_t)$**

This proof is identical to the proof of the follower growth option. Simply change $A^F_i$ to $A^S_i$, $B^F_i$ to $B^S_i$ and set $\kappa$ equal to zero.
Derivation of Leader option $O^L_i(X_t)$

$$O^L_i(X_t, X^F_j) = \mathbb{E}_t^Q \left[ \left( \int_{\tau^F_j}^{\infty} (A^L_i X_s - B^L_i) e^{-r(s-t)} ds \right) \right]$$

$$= \mathbb{E}_t^Q \left[ e^{-r(\tau^F_j-t)} \left( \int_{\tau^F_j}^{\infty} (A^L_i X_s - B^L_i) e^{-r(s-\tau^F_j)} ds \right) \right]$$

$$= \mathbb{E}_t^Q \left[ (e^{-r(\tau^F_j-t)}) \mathbb{E}_{\tau^F_j}^Q \left[ \left( \int_{\tau^F_j}^{\infty} (A^L_i X_s - B^L_i) e^{-r(s-\tau^F_j)} ds \right) \right] \right]$$

$$= \mathbb{E}_t^Q \left[ e^{-r(\tau^F_j-t)} \left( \frac{A^L_i X^F_j}{\delta} - \frac{B^L_i}{r} \right) \right]$$

$$= \left( \frac{X_t}{X^F_j} \right)^{\gamma} \left( \frac{A^L_i X^F_j}{\delta} - \frac{B^L_i}{r} \right).$$

Appendix C. Derivation of the conditions when firm 2 preempts or not

Firm 2 does not attempt to preempt when it has no incentive to be the leader. Formally, this requires that

$$\xi_2(x) \equiv V^L_2(x) - V^F_2(x)$$

is negative for all $x \in (0, X^F_2)$. Also for ease of notation, define $l_2 - l_1 \equiv l$. Therefore, in order to determine the domain of $l$ values where firm 2 does not preempt, we are interested in finding a pair $(x^{**}, l^{**})$ that satisfies the following system of equations:

$$\xi_2(x^{**}, l^{**}) = 0 \quad (C1)$$

$$\frac{\partial \xi_2(x, l^{**})}{\partial x} \bigg|_{x=x^{**}} = 0 \quad (C2)$$

In other words, we are interested in a point $(x^{**}, l^{**})$ at which firm 2’s leader function is tangent to the follower function. Substituting the firm values and after slight algebra, we have that $\xi_2(x)$
\[
\frac{A_F^2 - A_L^2}{\delta} x + \frac{B_L^2 - B_F^2}{r} - I_0 + \left( \frac{A_L^2 X_1^F}{\delta} - \frac{B_L^2}{r} \right) \left( \frac{x}{X_1^F} \right)^\gamma - \left( \frac{A_F^2 X_F^F}{\delta} - \frac{B_F^2}{r} \right) \left( \frac{x}{X_2^F} \right)^\gamma \bigg|_{x=x^{**}} = 0
\]  
(C3)

Differentiating with respect to \(x\) gives

\[
\frac{A_F^2 - A_L^2}{\delta} + \left( \frac{A_L^2 X_1^F}{\delta} - \frac{B_L^2}{r} \right) \left( \frac{x}{X_1^F} \right)^\gamma \left( \frac{\gamma}{x} \right) - \left( \frac{A_F^2 X_F^F}{\delta} - \frac{B_F^2}{r} \right) \left( \frac{x}{X_2^F} \right)^\gamma \left( \frac{\gamma}{x} \right) \bigg|_{x=x^{**}} = 0
\]  
(C4)

Multiplying equation (C4) by \(x/\gamma\), and subtracting from equation C3 yields

\[
\frac{A_F^2 - A_L^2}{\delta} x^{**} \frac{\gamma - 1}{\gamma} + \frac{B_L^2 - B_F^2}{r} - I_0 = 0.
\]

After slight algebra, we obtain:

\[
x^{**} = \frac{\gamma}{\gamma - 1} \frac{B_L^2 - B_F^2}{\frac{A_F^2 - A_L^2}{\delta}} + I_0 > 0.
\]  
(C5)

Substituting equation (C5) in equation (C3) yields and implicit equation for \(l^{**}\):

\[
\frac{B_L^2 - B_F^2}{r} + I_0 + \left( \frac{A_L^2 X_1^F}{\delta} - \frac{B_L^2}{r} \right) \left( \frac{x}{X_1^F} \right)^\gamma - \left( \frac{A_F^2 X_F^F}{\delta} - \frac{B_F^2}{r} \right) \left( \frac{x}{X_2^F} \right)^\gamma \bigg|_{x=x^{**}} = 0.
\]  
(C6)

If \(l > l^{**}\), then \(\xi(x^{**})\) is negative. This means that firm 2 does not prefer to be the follower in the domain \(x \in (0, X_2^F)\). Therefore, firm 1 invests at \(X_1^{LN}\) without fear of preemption by firm 2.
Appendix D. Derivation of the conditions when firm 1 simultaneously invests or becomes the leader

Firm 1 simultaneously invests unless the value from simultaneous investment is less than the value from being the leader. Formally, this requires that

\[ \zeta_1(x) \equiv V_1^L(x) - V_1^S(x) \]

is negative for all \( x \in (0, X_2^F) \). Therefore, in order to determine the domain of \( l \) values where firm 1 simultaneously invests with firm 2, we are interested in finding a pair \( (x^*, l^*) \) that satisfies the following system of equations:

1. \[ \zeta_1(x^*, l^*) = 0 \quad (D1) \]
2. \[ \frac{\partial \zeta_1(x^*, l^*)}{\partial x} \bigg|_{x = x^*} = 0 \quad (D2) \]

In other words, we are interested in a point \( (x^*, l^*) \) at which firm 1’s leader function is tangent to the simultaneous value function. Substituting the firm values and after slight algebra, we have that \( \zeta_1(x) \) equals

\[ \frac{A_1^S - A_1^L}{\delta} - \frac{B_1^L - B_1^S}{r} - I_0 + \left( \frac{A_1^L X_2^F}{\delta} - \frac{B_1^L}{r} \right) \left( \frac{x}{X_2^F} \right)^\gamma - \left( \frac{A_1^S X_1^S}{\delta} - \frac{B_1^S}{r} \right) \left( \frac{x}{X_1^S} \right)^\gamma \bigg|_{x = x^*} = 0 \quad (D3) \]

Differentiating with respect to \( x \) gives

\[ \frac{A_1^S - A_1^L}{\delta} + \left( \frac{A_1^L X_2^F}{\delta} - \frac{B_1^L}{r} \right) \left( \frac{x}{X_2^F} \right)^\gamma \left( \frac{\gamma}{x} \right) - \left( \frac{A_1^S X_1^S}{\delta} - \frac{B_1^S}{r} \right) \left( \frac{x}{X_1^S} \right)^\gamma \left( \frac{\gamma}{x} \right) \bigg|_{x = x^*} = 0 \quad (D4) \]

Multiplying equation (D4) by \( x/\gamma \), and subtracting from equation D3 yields

\[ \frac{A_1^S - A_1^L}{\delta} x^* \gamma - 1 \gamma + \frac{B_1^L - B_1^S}{r} - I_0 = 0. \]
After slight algebra, we obtain:

\[ x^* = \frac{\gamma}{\gamma - 1} \left( \frac{B_S^l - B_L^l}{r} \right) + I_0 > 0. \] (D5)

Substituting equation (D5) in equation (D3) yields an implicit equation for \( l^* \):

\[
\frac{B_S^l - B_L^l}{r} + I_0 + \left( \frac{A_l^L X_F^2}{\delta} - \frac{B_L^l}{r} \right) \left( \frac{x}{X_F^2} \right)^\gamma - \left( \frac{A_S^S X_S^1}{\delta} - \frac{B_S^S}{r} \right) \left( \frac{x}{X_S^1} \right)^\gamma \bigg|_{x=x^*} = 0. \] (D6)

If \( l < l^* \), then \( \zeta(x^*) \) is negative. This means that firm 1 does not prefer to be the leader. Therefore, firm 1 invests at \( X_1^S \).
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