Oligopoly Intermediation, Strategic Pre-Commitment and the Mode of Competition

By Stephen F. Hamilton and Philippe Bontems

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This paper considers oligopoly competition among market intermediaries who set prices jointly in upstream and downstream markets. We examine two forms of sequential pricing games first considered by Stahl (1988). The equilibrium outcomes of the model span the range between Bertrand and Cournot depending on the relative degree of product differentiation in the upstream and downstream markets. Prices are strategic complements when the oligopoly interaction is relatively balanced across the two markets, whereas prices are strategic substitutes when the degree of relative product differentiation is sufficiently high. We derive inferences for strategic pre-commitment in settings with vertical delegation and detail testable implications for the mode of competition that depend only on primitives of supply and demand.

An obstacle to deriving policy implications from the oligopoly model is the sensitivity of strategic pre-commitment devices to the mode of competition. Indeed, the strategic underpinnings of the oligopoly model depend fundamentally on the manner in which a firm’s choice variables alter the marginal profit expressions of their rivals, a feature first noted by Bulow, Geanakoplos, and Klemperer (1985) and Fudenberg and Tirole (1984). In the strategic trade policy literature, export subsidies are optimal when

* Hamilton: Department of Economics, Orfalea College of Business, California Polytechnic State University, San Luis Obispo. Email: shamilto@calpoly.edu. Bontems: Toulouse School of Economics (GREMAQ, INRA, IDEI). Email: bontems@toulouse.inra.fr. Part of this paper was written while Stephen F. Hamilton was visiting the Toulouse School of Economics and the author thanks members of that department for their hospitality. We are indebted to Jason Lepore for invaluable advise on an earlier draft of the paper, and thank Robert Innes, Richard Sexton, and seminar participants at UC Berkeley and the University of Kiel for helpful comments
firm’s choice variables are strategic substitutes (Brander and Spencer, 1985), but export
taxes are optimal when firm’s choice variables are strategic complements (Eaton and
Grossman, 1986). In the strategic delegation literature, the optimal managerial contract
overcompensates sales when agents compete in strategic substitutes, but overemphasizes
profits when agents compete in strategic complements (Fershtman and Judd, 1987; Skli-
vases 1987). In the vertical separation literature, the optimal contract involves retailers
charging “sloting allowances” for shelf-space when competing in strategic complements
(Shaffer, 1991), but involves negative payments for shelf space when firms compete in
strategic substitutes (Vickers, 1985).\(^1\) Characterizing and measuring the mode of com-
petition in industrial settings is essential to formulate effective policies under oligopoly.

In this paper, we construct an oligopoly framework that generates testable hypotheses
on the mode of competition in industrial settings. We frame our model around duopoly
firms who serve as intermediaries between an upstream market and a downstream market.
The products in each market are differentiated, as would be the case when manufacturers
rely on specialized inputs to produce consumer goods, and resources are constrained in
the upstream input market in the sense that devoting resources to the production of the
input for one firm reduces resources available to produce the input for the other firm.

Our approach extends the analysis of Stahl (1988) to differentiated product markets.
Stahl (1988) considers a setting of oligopoly intermediation with homogeneous products
and “winner-take-all” competition under two forms of sequential price competition: \((i)\)
“purchasing to stock” (PTS), in which firms select input prices prior to setting output
prices; and \((ii)\) “purchasing to order”, (PTO), in which firms sell forward contracts to
consumers prior to selecting input prices. In each case the market outcome is Bertrand,
and Walrasian prices arise from the optimal price-setting behavior of firms. Our point of
departure is to introduce product differentiation on each side of the market, and doing so
creates essential differences between PTS and PTO outcomes according to the “relative
degree of product differentiation” in the upstream and downstream markets.

\(^1\)The policy ambiguity of the oligopoly model dates back to the original oligopoly models of Cournot (1838) and
Bertrand (1883) and extends to sequential games. It has been known since von Stackelberg (1934) that advantage goes
to the “first-mover” when choice variables are strategic substitutes, but more recently recognized that being first-mover is
dis disadvantageous in the case of strategic complements (Gal-Or, 1985).
The relative degree of product differentiation is a measure of the relative strength of the oligopoly interaction in the upstream and downstream markets in which the firms compete. We define it to be the difference between the ratio of the cross-price to own-price elasticity of supply and the ratio of cross-price to own-price elasticity of demand. We show that the Pareto dominant equilibrium for firm profits is PTS when products in the upstream market are relatively highly differentiated and PTO when products in the downstream market are relatively highly differentiated. More importantly, our analysis allows us to characterize the mode of competition quite precisely according to commonly-estimated supply and demand conditions. For both the PTS and PTO equilibrium, we demonstrate that prices are strategic substitutes when the degree of relative product differentiation is sufficiently high, while prices are strategic complements for more balanced degrees of oligopoly interaction.

We connect our model to the oligopoly literature by deriving conditions under which the intermediated oligopoly equilibrium results in Bertrand and Cournot outcomes. When the upstream and downstream markets are equally differentiated, the Pareto dominant equilibrium of the oligopoly model is Bertrand. The intermediated oligopoly model thus produces an envelope of Bertrand outcomes with different degrees of absolute product differentiation, among which one is the Walrasian equilibrium examined by Stahl (1988). Alternatively, when firms serve monopsony upstream markets, PTS is the Pareto dominant equilibrium and the resulting outcome is Cournot oligopoly. In this case, purchasing inputs from an upstream market prior to selecting output prices has the character of a capacity commitment, and it is has been known since Kreps and Scheinkman (1983) that capacity commitments under price competition result in Cournot outcomes. Our model extends this result from the case of fixed quantity pre-commitments (vertical supply functions) under capacity choices to encompass general supply functions derived from monopsony upstream markets. The converse is also true. When the firms serve independent monopoly downstream markets, PTO is the Pareto dominant equilibrium and the resulting outcome is Cournot oligopsony.

The essential features of the oligopoly intermediation model can be illustrated by
contrasting the PTO equilibrium with the Bertrand model of price competition. In the Bertrand model, inputs are taken as exogenous when selecting prices for the downstream consumer market, whereas firms in the PTO model consider the effect of changes in their output prices on the input prices paid in the upstream market. Under circumstances where PTO is the dominant equilibrium, an increase in a firm’s output price drives down his rival’s input price along the rival’s supply function, which serves to soften price competition jointly in both markets. PTO thus facilitates higher equilibrium output prices—and lower equilibrium input prices—than would arise under Bertrand.

We believe it is a natural extension of the oligopoly model to consider the strategic interdependence of firms’ choice variables at all points of contact with their rivals. Firms in a given industry sell similar products to consumers, but also procure similar stocks from suppliers. Our model closes the loop in the circular flow of factors and goods between consumers and firms by allowing the oligopoly interaction with rivals to occur jointly in upstream and downstream markets. Doing so generates testable hypotheses on the mode of competition that depend entirely on primitives of the supply and demand functions facing firms.

Previous research by Klemperer and Meyer (1989) and Maggi (1996) has endogenized the mode of competition in the oligopoly model. In Klemperer and Meyer (1989), firms choose supply functions, which allows them to adapt to market outcomes under uncertainty. In Maggi (1996) firms invest in capacity subject to *ex post* adjustment costs that are capable of relaxing their capacity commitments. The sequential adjustment process in these models, which is similar to the sequential pricing structure that underlies our model, produces a continuum of outcomes that span the modes of competition. Our oligopoly intermediation model has advantages over these approaches in terms of simplicity and empirical content.

We apply our model to derive observations on the type of policy circumstances that favor alternative forms of strategic pre-commitment. In the case of strategic international trade, export subsidization is optimal when the relative degree of product differentiation is sufficiently high. When products in the upstream market are relatively highly dif-
differentiated, the optimal trade policy is oriented towards rent-shifting in the downstream international market. Export subsidies emerge as a strategic trade policy when the degree of oligopoly interaction is sufficiently small in the upstream market, for instance when a domestic farm input is used to produce a traded agricultural good. Export subsidies also emerge when the downstream market is relatively highly differentiated, as would occur when firms procure commoditized inputs from international suppliers such as components for consumer electronics. Taxes are optimal as a strategic trade policy when the intensity of the oligopoly interaction with foreign rivals is relatively balanced in upstream and downstream markets; however, the natural strategic trade policy instrument in this case is a tariff on the imported input. Tariffs on imported inputs can substitute for export taxes as a strategic trade policy instrument in the intermediated oligopoly model and are more commonly observed than export taxes in international trade settings.

Our model produces a unique strategic pre-commitment policy for any set of supply and demand conditions facing oligopoly firms. We numerically solve for the optimal policy in the case of linear supply and demand functions and plot the outcomes for variations in the degree of product differentiation in the downstream market. The optimal policy under all forms of strategic delegation follows an inverted u-shaped pattern as products in the downstream market become more highly differentiated. In the case of managerial incentives, for example, the optimal contract over-emphasizes sales for sufficiently low levels of downstream product differentiation, switches to a policy that over-emphasizes profits for a range of market outcomes with relatively equal degrees of product differentiation, and then reverts again to a policy that over-emphasizes sales for a sufficiently high degree of downstream product differentiation. A qualitatively identical pattern emerges for strategic delegation in international trade policy and vertical separation.

The remainder of the paper is structured as follows. In the next Section we present the model and characterize the Bertrand, PTS and PTO equilibria. In Section 3, we derive our main results by classifying Pareto dominant Nash equilibrium and characterizing the mode of competition in the oligopoly model according to the relative degree of product differentiation in the upstream and downstream markets. In Section 4, we present
an example with linear supply and demand that results in closed-form expressions for equilibrium prices and we use this example to numerically illustrate our main findings. In Section 5, we consider applications of the model to pre-commitment policies under various forms of vertical delegation and Section 6 concludes. We collect all the proofs of our Propositions in the Appendix.

I. The Model

We consider duopoly intermediaries, labeled firms 1 and 2, who compete against each other in prices. The firms purchase differentiated stocks (“inputs”) from suppliers in an upstream market, and sell finished products (“outputs”) derived from the inputs to consumers in a downstream market. Both input suppliers and consumers are price-taking agents in their respective markets.

To clarify the outcomes pertaining to the oligopoly model, we consider fixed proportions technology. Specifically, letting $x_i$ denote the quantity of the input purchased in the upstream market by firm $i$, we scale units such that $y_{iD} = x_i$ denotes the quantity of the output sold by the firm in the downstream market. Products in each market are differentiated, and the degree of oligopoly interaction between the intermediaries potentially differs at each point of contact between the firms. One interpretation of the model is that demand and supply functions depend on the spatial location of suppliers, consumers, and firms. Another interpretation is that the firms require specialized inputs to produce customized outputs.

Let $p = (p_1, p_2)$ denote the vector of output prices. Consumer demand for (differentiated) product $i$ is given by

\[ D_i^j = D_i (p), \quad i = 1, 2, \]

where $D_i^j = \partial D_i / \partial p_i < 0$, and $D_j^j = \partial D_i / \partial p_j \geq 0$, with the latter condition serving to confine attention to the case of substitute goods.\(^2\) Let $w = (w_1, w_2)$ denote the vector $\implies$\(^2\)Our model readily extends to the case of complements. As in Singh and Vives (1984), the outcome with complementary goods is a mirror image of the outcome with substitute goods.
of input prices, so that the supply function facing firm $i$ in the upstream market is

\begin{equation}
S^i = S^i(w), \quad i = 1, 2,
\end{equation}

where $S^i = \frac{\partial S^i}{\partial w_i} > 0$, and $S^j = \frac{\partial S^j}{\partial w_j} \leq 0$ (i.e., products in the upstream market are substitutes). Throughout the paper, we assume that the direct effect of a price change outweighs the indirect effect in each market; that is, $\Delta \equiv D^i_j D^j_i - D^j_i D^i_j > 0$ and $\Sigma \equiv S^i_i S^i_j - S^j_i S^j_i > 0$. These conditions are sufficient for stability of the equilibrium (see, e.g., Vives, 1999).\(^3\)

We suppress inventory-holding behavior and the destruction or removal of goods. Accordingly, the demand and supply functions facing each firm are linked by the material balance equation,

\begin{equation}
y_i = D^i(p) = S^i(w), \quad i = 1, 2
\end{equation}

where $y_i$ is the market-clearing quantity for intermediary $i$ in each market.

The concept of relative product differentiation is important for the analysis to follow. The relative degree of product differentiation is a measure of the relative strength of the oligopoly interaction in the upstream and downstream markets in which the firms compete. Focusing on the symmetric market equilibrium, we describe the relative degree of product differentiation in terms of market supply and demand elasticities. Specifically, let $\epsilon_s = \frac{e^i_{ii}}{e^i_{ij}} < 1$ denote the ratio of supply elasticities in the upstream market, where $e^i_{ii} = S^i_i w / S^i_i > 0$ is the own-price elasticity of supply and $e^i_{ij} = -S^i_j w / S^i_i > 0$ is the cross-price elasticity of supply, and let $\epsilon_d = \frac{e^d_{ii}}{e^d_{ij}} < 1$ denote the ratio of demand elasticities in the downstream market, where $e^d_{ii} = -D^i_i p / D^i_i > 0$ and $e^d_{ij} = D^j_i p / D^j_i > 0$ are the own- and cross-price elasticities of demand. This results in the following:

**Definition 1:** $\Theta = \epsilon_s - \epsilon_d$ describes the degree of product differentiation in the downstream market relative to the upstream market.

\(^3\)These conditions also ensure that the system of demand and supply equations are invertible.
We refer to the downstream market as being relatively more differentiated than the upstream market when $\Theta > 0$, the upstream market as being relatively more differentiated than the downstream market when $\Theta < 0$, and the markets as being equally differentiated when $\Theta = 0$.

We consider various settings of oligopoly competition: (i) purchasing to stock (PTS), in which firms set input prices prior to setting consumer prices; (ii) purchasing to order (PTO), in which firms sell forward contracts to consumers and then procure inputs necessary to fill them in the upstream market, and (iii) simultaneous price-setting behavior. While our main interest is on the PTS or PTO outcomes, we consider the Bertrand equilibrium of simultaneous price-setting behavior as a convenient benchmark for the analysis to follow.

A. Bertrand Outcomes

Consider first the case in which firms select prices simultaneously in the upstream and downstream markets. Firm $i$ seeks to maximize profits

\begin{equation}
\pi^{i,B}(p, w) = p_i D^i(p) - w_i S^i(w) - F_i, \quad i = 1, 2
\end{equation}

subject to the inventory constraint, $S^i(w) \geq D^i(p), i = 1, 2$. Confining attention to interior solutions, the Bertrand equilibrium satisfies

\begin{equation}
p_i - w_i = \frac{S^i(w)}{S^i(w)} - \frac{D^i(p)}{D^i(p)}, \quad i = 1, 2
\end{equation}

Simultaneously solving (5) subject to the inventory constraint (3) gives the equilibrium prices, which we define in the symmetric case as $(w^B, p^B)$. The equilibrium price-cost margin for symmetric firms can be written

\begin{equation}
p^B - w^B = \frac{p^B}{e_{ii}^o} + \frac{w^B}{e_{ii}^e}.
\end{equation}

Notice in equation (6) that the equilibrium price-cost margin depends jointly on the own-
price elasticities of supply and demand. Specifically, the Lerner index of market power, 
\[ L^B = \frac{p^B - w^B}{p^B} \]
is
\[ L^B = \frac{\frac{1}{\epsilon_{ii}} + \frac{1}{\epsilon_{ii}}}{1 + \frac{1}{\epsilon_{ii}}} \].
The outcome under intermediated oligopoly differs from the Bertrand outcome with exogenous inputs, which results in the familiar rule that 
\[ L = \frac{1}{\epsilon_{ii}} \]. The reason is that Bertrand intermediaries select prices simultaneously in both markets, so that the degree of market power exercised by firms rises in the oligopoly equilibrium with less less price-elastic supply conditions.

B. Purchasing to Stock (PTS)

Under purchasing to stock (PTS), firms first choose input prices to acquire stocks in the upstream market, and subsequently select consumer prices in the downstream market. We denote the vector of input prices set by the firms as \( w = (w_1, w_2) \). The input prices determine the quantity of the input purchased by each firm according to the supply functions (2), and this in turn implies a unique quantity of output for each firm, 
\[ y_i = S'(w), i = 1, 2 \], with the associated output price vector \( p = (p_1, p_2) \) defined by equations (1) and (3).

Given the input price vector \( w = (w_1, w_2) \) selected under PTS, firm \( i \) inherits a quantity that clears the upstream market, \( \bar{y}_i = S'(w) \). Provided that the demand function for firm \( i \) is monotonically decreasing, it follows from equation (3) that \( \bar{y}_i = D'(p_1, p_2) \), which implicitly defines the function 
\[ p_i \equiv p_i(p_j | \bar{y}_i) \text{ for } i = 1, 2 \]. Equating these functions yields the retail market equilibrium condition for firm \( i \),

\[ p_i \equiv p_i(\bar{y}_1, \bar{y}_2) = p_i(y(w)), \]
where \( y(w) = (y_1(w), y_2(w)) \) is the quantity vector defined by the first-stage wholesale pricing equilibrium. Denote the vector of equilibrium inverse demands by \( p(w) = (p^1(w), p^2(w)) \).
The slopes of inverse demand function $i$ with respect to input prices $i$ and $j$ are

$$
\frac{\partial p_i'(w)}{\partial w_i} = \frac{\partial p_i'(y(w))}{\partial y_i} \frac{\partial y_i'(w)}{\partial w_i} + \frac{\partial p_i'(y(w))}{\partial y_j} \frac{\partial y_j'(w)}{\partial w_i},
$$

$$
\frac{\partial p_j'(w)}{\partial w_j} = \frac{\partial p_j'(y(w))}{\partial y_i} \frac{\partial y_i'(w)}{\partial w_j} + \frac{\partial p_j'(y(w))}{\partial y_j} \frac{\partial y_j'(w)}{\partial w_j},
$$

respectively. The slopes of the inverse demand function for firm $i$ with respect to the input prices can be characterized as follows. First, by definition, $S_i'(w) \equiv \frac{\partial y_i'(w)}{\partial w_i}$ and $S_j'(w) \equiv \frac{\partial y_j'(w)}{\partial w_j}$ are the supply-side effects of an input price change on the quantity of inputs procured by firm $i$. Next, recall that both firm $i$ and firm $j$ set output prices subject to the quantity produced from their input purchases in the upstream market. From the perspective of firm $i$, the rival firm $j$ competes in output prices according to the fixed quantity level $y_j(w) = y_j$ defined by the previous input pricing stage. This implies that firm $j$’s reaction function is implicitly defined by $y_j(p) = y_j$ in the downstream market, such that for a constant quantity sold by firm $j$, residual demand for firm $i$ is given by $y_i' \equiv D_i'(p_i, p_j(p_i))$, where $p_j(p_i)$ represents the output price adjustment that firm $j$ must make to maintain $y_j$ for the output price selected by firm $i$. Thus,

$$
\frac{\partial y_j'}{\partial p_i} \bigg|_{y_j = y_j} = D_j' + D_j' \left( \frac{\partial p_j}{\partial p_i} \bigg|_{y_j = y_j} \right).
$$

Totally differentiating the demand function of firm $j$ gives $\frac{\partial p_j}{\partial p_i} \bigg|_{y_j = y_j} = \frac{-D_j'}{D_j'}$. Substituting this value into equation (10) and inverting the resulting expression yields

$$
\frac{\partial p_i'(y(w))}{\partial y_i} = \frac{D_j'}{\Delta}.
$$

Proceeding similarly for the rival firm, the residual demand function facing firm $j$ is
given by $y_j = D_j^i \left(p^i(p_j), p_j\right)$, so that

$$\frac{\partial p^i(y(w))}{\partial y_j} = -\frac{D_j^i}{\Delta}.$$ 

(12)

Making use of (11) and (12) in (8) and (9) and simplifying the resulting expressions gives

$$p^i_j(w) = \frac{D_j^i S_i^j - D_j^i S_j^i}{\Delta} < 0$$

(13)

$$p^j(w) = \frac{D_j^i S_j^i - D_j^i S_j^j}{\Delta}.$$ 

(14)

Equation (13) represents the own-price effect of an increase in the input price on the output price of the firm. This term is negative under our stability conditions. When a firm raises his input price, a greater quantity of the input is acquired, and this drives down his output price by narrowing the firm’s price-cost margin.

Equation (14) is the cross-price effect of a input price increase on the output price of the rival firm. The sign of this term is important for the results to follow. Firm $i$ responds to an input price increase of his rival ($\partial w_j > 0$) by increasing his output price ($\partial p_i > 0$) whenever $D_j^i S_i^j > D_j^i S_j^j$ and otherwise decreases his output price.

When a firm changes his input price, this creates two offsetting effects on the output price of his rival. First, selecting a higher input price bids stocks away from the rival in the upstream market. This “supply effect” reduces the output level of the rival and places upward pressure on the rival’s output price. Second, the combined procurement level of the firms increases (rival’s input price given) following a unilateral rise in the firm’s input price. Total output rises, and this “demand effect” floods the downstream market with output, dampening the output price of the rival. In general, which of these two effects is dominant depends on the relative degree of product differentiation of the downstream market. If the products in the downstream market are nearly homogeneous,
the demand effect dominates the supply effect, and an input price increase by one firm that raises total output must reduce the output price of the rival. Conversely, if firms sell their output in segmented monopoly downstream market(s), the demand effect vanishes entirely and the supply effect must dominate: The rival responds to an increase in the input price by selecting a higher output price.

In the symmetric market equilibrium \( (D^i = D^j, S^i = S^j, p_1 = p_2 = p, w_1 = w_2 = w) \), this condition can be expressed by making use of our definition of relative product differentiation as

\[ p_i^j(w) = \Theta, \]

where “\( \doteq \)" denotes “equal in sign”. Under circumstances in which the upstream market is relatively more differentiated than the downstream market (\( \Theta < 0 \)), a rise in the input price by a firm decreases the output price set by his rival.

With homogeneous products in each market, \( \Theta = 0 \), the input price choice of one firm has no implications for the retail price set by his rival. This property holds in the intermediated oligopoly equilibrium considered by Stahl (1988).

Next consider the selection of input prices under PTS. Firm \( i \) seeks to maximize profits,

\[ \pi^{i,s}(w_i, w_j, F_i) = (p_i^i(w) - w_i) S_i'(w) - F_i, \quad i = 1, 2, \]

where \( p_i^i(w) \) is given by (7) and \( F_i \) denotes fixed costs, a portion of which may be sunk. The first-order necessary condition for a profit maximum is

\[ (15) \quad \pi_i^{i,s} = (p_i^i(w) - w_i) S_i'(w) + (p_i^j(w) - 1) S_i'(w) = 0, \quad i = 1, 2, \]

where \( p_i^j(w) < 0 \) is as defined in (13).

The equilibrium under PTS is determined by the simultaneous solution of equations (15). Define the equilibrium input price vector that solves these equations by \( w^* = (w_1^*, w_2^*) \). Substitution of input prices into the supply functions yields the outputs, \( y^* = (y_1^*, y_2^*) \), and the resulting equilibrium output prices, \( p^* = (p_1^*, p_2^*) \).
The equilibrium price-cost margin for symmetric firms can be written

\[ p^s - w^s = k^s \frac{P^s}{e_{ii}^s} + \frac{w^s}{e_{ii}^s}, \]

where \( k^s = \frac{1 - \epsilon_{e_{ii}}}{1 - \epsilon_{e_{ii}}} \). Notice that the second term on the right-hand side of equation (16) is identical to the second term on the right hand-side of equation (6). If this was the only term that decided equilibrium prices, then the outcome would be Bertrand. The first term on the right-hand side of equation (16) introduces a weight on the “demand-side” portion of the equilibrium price-cost margin that jointly accounts for the degree of product differentiation in both the upstream and downstream markets. It is straightforward to verify that \( k^s - 1 \equiv -\Theta \). Firm select higher price-cost margins in the PTS equilibrium than in the Bertrand equilibrium when the upstream market is relatively more differentiated than the downstream market (\( \Theta < 0 \)). The intuition for this is quite clear: When \( \Theta < 0 \), a decrease in the input price by a firm increases the output price of his rival, thereby softening price competition in the downstream market.

The Lerner index of market power under PTS is given by

\[ L^s = \frac{k^s + \frac{1}{e_{ii}^s}}{1 + \frac{1}{e_{ii}^s}}. \]

We formally characterize equilibrium strategies in Section 3, which allows for a more complete comparison of PTS and Bertrand outcomes. Before turning to this analysis, we derive the market equilibrium in the remaining case of PTO.

C. Purchasing to Order (PTO)

Suppose the firms sell forward contracts for delivery of consumer goods prior to procuring stocks in the upstream market. Forward contracts are widely used in practice, including imported goods sold to retail distributors and a significant portion of wholesale trade.

In the input pricing stage, firm \( i \) sets prices to meet his forward contracts with con-
sumers in the downstream market. Defining this quantity to be \( y_i \) for the output price vector \( p = (p_1, p_2) \) and proceeding as before by equating the implicit functions, \( w_i \equiv w^i (y_j) \), \( i = 1, 2 \), the input pricing condition for firm \( i \) is

\[
w_i = w^i (y_1, y_2) = w^i (y(p)) ,
\]

where \( y(p) = (y_1(p), y_2(p)) \) is the quantity vector defined by the first-stage selection of output prices. Denote the vector of inverse supply functions by \( w = (w^1, w^2) \).

The slopes of (inverse) supply function \( i \) with respect to output prices \( i \) and \( j \) are

\[
w^i_j = \frac{\partial w^i(p)}{\partial p_i} = \frac{\partial w^i(y(p))}{\partial y_i} \frac{\partial y^i(p)}{\partial y_j} + \frac{\partial w^i(y(p))}{\partial y_j} \frac{\partial y^i(p)}{\partial p_i} < 0 ,
\]

respectively. Inverting the supply functions and making the corresponding substitutions gives

\[
w^i_j(p) = \frac{D_i^j S^i_j - D_j^i S^i_j}{\Sigma} < 0
\]

Equation (18) represents the effect of an increase in the output price on the firm’s own input price and equation (19) is the cross-price effect of a output price change by the rival on the firm’s input price.

Notice that the cross-market effect in equation (19) under PTO always takes the opposite sign of the cross-market effect in equation (14) under PTS; that is, \( p^j_i(w) = -w^i_j(p) \).

Under market conditions in which an increase in the input price increases (decreases) the output price of the rival under PTS, an increase in the output price decreases (increases) the input price of the rival under PTO. In the symmetric market equilibrium
When a firm increases his output price, demand shifts out for the rival, which causes the rival’s input price to be bid upward along his supply function to meet the increase in customer orders. This is the demand effect. Nevertheless, the combined quantity sold by the firms decreases (rival’s output price given) following a unilateral rise in the firm’s output price, and this supply effect places downward pressure on the rival’s input price.

As in the case of PTS, the relative degree of product differentiation is again essential for determining the strategic response of the rival. If the products in the downstream market are nearly homogeneous, the demand effect dominates the supply effect, and a consumer price increase by one firm raises the input price set by both firms. If the products in the downstream market are monopoly goods, there is no demand effect, and the supply effect dominates: An increase in the output price by one firm reduces the input price of the rival.

In the output pricing stage, firm \( i \) acquires forward contracts to maximize profits of

\[
\pi^i_o(p_i, p_j, F_i) = (p_i - w^i(p)) D^i(p) - F_i, \quad i = 1, 2,
\]

where \( w^i(p) \) is given by equation (17). The first-order necessary condition for a profit maximum is

\[
(20) \quad \pi^i_{1,o} \equiv (p_i - w^i(p)) D^i(p) + (1 - w^i(p)) D^i(p) = 0, \quad i = 1, 2,
\]

where \( w^i(p) < 0 \) is as defined in equation (18).

The equilibrium under PTO is determined by the simultaneous solution of equations (20). Define the equilibrium output price vector that solves these equations as \( p^o = (p_1^o, p_2^o) \) and the associated vector of equilibrium input prices as \( w^o = (w_1^o, w_2^o) \).
The equilibrium price-cost margin for symmetric firms can be written

\[ p^o - w^o = \frac{P^o}{v^o_{ij}} + k^o \frac{W^o}{v^o_{ij}}. \]

where \( k^o = \frac{1 - \epsilon_2 \epsilon_i}{1 - \epsilon_2} \). Visual inspection of this term reveals that \( k^o > 1 \) when \( \Theta > 0 \). Firms set higher price-cost margins in the PTO equilibrium than in the Bertrand equilibrium when the downstream market is relatively more differentiated than the upstream market \((\Theta > 0)\). The reason is that a rise in the output price by a firm decreases the input price set by his rival when \( \Theta > 0 \), facilitating higher price-cost margins.

The Lerner index of market power under PTO is given by

\[ L^o = \frac{1 + \epsilon_2}{1 + \epsilon_2}. \]

II. Equilibrium Outcomes

In this Section we identify Pareto dominant strategies in the symmetric market equilibrium. To do so, we consider the symmetric Bertrand equilibrium \( (w^B, p^B) \) and examine multilateral defections from the equilibrium that increase the profits of firms.\(^4\)

A. PTS Versus PTO

Consider first the case of PTS competition. Evaluating the input price condition (15) at the symmetric Bertrand equilibrium position \( (w^B, p^B) \) gives

\[ \pi_i^{i,s}(w^B, p^B) \equiv \left( \frac{D^i_i S^i_i - D^i_j S^i_j}{\Delta} - 1 \right) S^i - D^i \left( \frac{S^i_i}{D^i_i} \right). \]

Making use of the market-clearing condition (3) and factoring terms yields

\[ \pi_i^{i,s}(w^B, p^B) \equiv \frac{S^i D^i S^i_i}{\Delta} \Theta \equiv \Theta. \]

Under PTS, firms select higher input prices than in the symmetric Bertrand equilibrium when \( \Theta > 0 \) and select lower input prices when \( \Theta < 0 \).

\(^4\)With slight abuse of notation, we write demand, supply, and profit as functions of the scalar values of input and output prices in the symmetric equilibrium.
Proceeding similarly in the case of PTO, evaluating the output price condition (20) at the symmetric Bertrand equilibrium position \((w^B, p^B)\) gives

\[
\pi_i^B(w^B, p^B) = \frac{D_i^j S_j^i}{\sum} \Theta = \Theta.
\]

Under PTO, firms set higher output prices than in the symmetric Bertrand equilibrium when \(\Theta > 0\) and lower output prices when \(\Theta < 0\).

In Stahl’s (1988) model, intermediaries compete in homogeneous product markets \((\Theta = 0)\), and a Walrasian equilibrium emerges under both PTO and PTS. Here, we extend this outcome of Bertrand merchants to encompass any market that satisfies \(\Theta = 0\). Thus, we arrive at

**PROPOSITION 1:** If \(\Theta = 0\) the equilibrium market outcome under either PTO or PTS is Bertrand.

The Bertrand equilibrium represents an envelope of oligopoly outcomes with equal degrees of product differentiation in the upstream market and downstream market. The essential underpinning of the Bertrand outcome is the independence between a firm’s input (output) price choice and the resulting output (input) price selected by his rival, and what matters for this outcome is the relative degree (rather than the absolute degree) of product differentiation. Bertrand merchants arise, even in the case of differentiated products, provided the merchants themselves do not contribute to product differentiation by specializing their input requirements or by customizing their outputs.

Given a value of \(\Theta \neq 0\), firms do not serve as Bertrand merchants under PTS or PTO. The equilibrium prices diverge from Bertrand levels in both cases, and the Pareto dominant equilibrium can be classified in terms of the relative degree of product differentiation as follows:

**PROPOSITION 2:** If \(\Theta > 0\) \((< 0)\) the Pareto dominant equilibrium is PTO (PTS).

Proposition 2 is highly intuitive. Profits are larger in the symmetric equilibrium when firms have wider price-cost margins. When \(\Theta < 0\), switching to PTS facilitates this
outcome, because the rival responds to a lower input price by selecting a higher output price in equation (14), thereby softening price competition in the downstream market. Equilibrium price-cost margins and profits are accordingly higher under PTS than under Bertrand. When $\Theta > 0$, switching to PTO is desirable, as the rival now responds to a higher output price by setting a lower input price in equation (19). Higher output prices in the downstream market soften input price competition in the upstream market, resulting in higher price-cost margins and profits under PTO. In either case, the dominant strategy of firms involves anchoring prices in their relatively more differentiated market.

B. Cournot Outcomes

The anatomy of the intermediated oligopoly model can be illustrated by describing the circumstances in which PTS and PTO competition produce Cournot outcomes. Under Cournot competition, firms compete in quantities, and the vector of quantities selected by the firms generates a unique set of market-clearing prices in the upstream and downstream markets.

Let $\mathbf{y} = (y_1, y_2)$ denote the vector of retail (and wholesale) quantities under Cournot competition. Defining inverse demand and inverse supply for product $i$ as $P^i(\mathbf{y})$ and $W^i(\mathbf{y})$, respectively, the profit of firm $i$ under Cournot competition is given by

$$\pi^{i,C}(y_i, y_j, F_i) = (P^i(\mathbf{y}) - W^i(\mathbf{y})) y_i - F_i.$$ 

The first-order necessary condition for a profit maximum is

$$P^i - W^i + \left( \frac{\partial P^i}{\partial y_i} - \frac{\partial W^i}{\partial y_i} \right) y_i = 0. \tag{21}$$

Simultaneously solving equations (21) in the symmetric equilibrium gives the equilibrium quantity, $y^C$, which can be used to recover the Cournot equilibrium prices ($w^C, p^C$).

The PTS and PTO equilibrium produce Cournot outcomes as follows:

PROPOSITION 3: The Cournot equilibrium emerges when $\epsilon_s = 0$ or $\epsilon_d = 0$. 

The PTS (PTO) equilibrium produces Cournot oligopoly (oligopsony) outcomes in the case of monopsony upstream (monopoly downstream) markets. To see why, consider the equilibrium price-cost margins for Cournot firms in the symmetric market equilibrium, which can be written, after making use of equations (13) and (18), as

\[
\frac{p^C - w^C}{\epsilon_{ii}} = \frac{p^C}{\epsilon_{ii} (1 - \epsilon_{ii}^2)} + \frac{w^C}{\epsilon_{ii} (1 - \epsilon_{ii}^2)}.
\]

Cournot intermediaries jointly consider the effect of a quantity increase on raising their procurement cost in the upstream market and on reducing their sales revenue in the downstream market. That is, the quantity selected by each firm pins down both the quantity supplied and quantity demanded simultaneously. In contrast, firms in the PTS (PTO) equilibrium, who set prices sequentially in the markets, can consider only one side of this interaction. When the oligopsony interaction between firms is replaced by monopsony upstream markets, \(s_D = 0\), the only interaction that remains with the rival is in the downstream market. The second term on the right-hand side of equation (22) reduces to \(\frac{w^C}{\epsilon_{ii}}\). Under PTS, which is the Pareto dominant equilibrium when \(\epsilon_s = 0\), the weight on the demand-side portion of the equilibrium price-cost margin in equation (16) reduces to \(k^s = \frac{1}{1-\epsilon_d^s}\) and the outcome is Cournot oligopoly. The PTS and Cournot outcomes are isomorphic when \(\epsilon_s = 0\), because the inability of firms to account for the effect of an output price change on the rival’s input pricing behavior under PTS no longer has any consequence. For a similar reason, PTO results in the Cournot oligopsony outcome with monopoly downstream markets, \(\epsilon_d = 0\).

We are now ready to show how the relative degree of product differentiation in the downstream market determines the mode of competition under oligopoly.

C. Mode of Competition

Characterizing and measuring the mode of competition in the oligopoly model is essential for deriving policy prescriptions in settings with strategic pre-commitment. It is also important for deriving inferences on the type of market conditions that warrant an-
titrust scrutiny, for instance market features that favor the use of slotting allowances as a facilitating practice. Our goal in this section is to develop testable hypotheses on the mode of competition in industrial settings. These implications can be made most clearly by focusing our attention on the first-order effects of the model that frame the use of linear estimation techniques.

To characterize the mode of competition in intermediated oligopoly settings, consider the second partials of profit under PTS and PTO. Dropping arguments for notational convenience, the effect of an input price change on the marginal profit of firm $i$ under PTS is

$$
\pi_{ii}^{i,s} \equiv (p_i - w_i)S_i + 2(p_i - 1)S_i^d + p_{ii}S_i < 0,
$$

and

$$
\pi_{ij}^{i,s} \equiv (p_i - w_i)S_j + (p_j - 1)S_j^d + p_{ij}S_j^d + p_{ij}S_i,
$$

where $\pi_{ii}^{i,s} < 0$ and $\pi_{ii}^{j,s} - \pi_{ij}^{i,s} > 0$. Input prices may be strategic complements ($\pi_{ij}^{i,s} > 0$) or strategic substitutes ($\pi_{ij}^{i,s} < 0$) in equation (23).

To provide an intuitive characterization of these outcomes, consider the first-order effects in expression (23). On substitution of equations (13) and (14), the mode of competition can be expressed as

$$
\pi_{ij}^{i,s} = \left( \frac{D_j^i S_j^i - D_j^i S_i^d}{\Delta} - 1 \right) S_j^d + \left( \frac{D_j^i S_j^i - D_j^i S_i^d}{\Delta} \right) S_i^d.
$$

In the symmetric market equilibrium, this condition can be written as

$$
\pi_{ij}^{s} \equiv \epsilon \left[ 1 + \frac{(p - w)}{p} \epsilon_{ii} \left( 1 - \epsilon_i^2 \right) \right] + \Theta,
$$

where the first term on the right hand side of equation (24) is positive and $\Theta < 0$ is

---

5 The Federal Trade Commission (FTC), which regulates the grocery industry, refused to provide guidelines for slotting allowances, citing the need for further investigation on the efficiency effects of the practice (FTC 2001).

6 We present results for CES and nested-Logit estimation models in the Web Appendix.
negative under PTS by Proposition 2.

The relative degree of product differentiation is important in determining the mode of competition under PTS. Indeed, it can be seen by inspection of terms in equation (24) that prices are strategic complements in the case of equal degrees of product differentiation, \( \Theta = 0 \) (the Bertrand equilibrium), and prices are strategic substitutes in the case of uncorrelated supply markets, \( \epsilon_s = 0 \) (the Cournot equilibrium). In general, \( \Theta < 0 \) under PTS is a necessary but not a sufficient condition for prices to be strategic substitutes.

Turning to the PTO equilibrium, we have

\[
\pi_{ij}^{i,o} = (p_i - w^i)D_{ij} + 2(1 - w^i)D_i - w^iD^i \in (-1, 0),
\]

and

\[
(25) \quad \pi_{ij}^{i,o} = (p_i - w^i)D_{ij} + (1 - w^i)D_j - w^iD_i - w^iD^i,
\]

where \( \pi_{ij}^{i,o} < 0 \) and \( \pi_{ij}^{i,o} \pi_{ij}^{j,o} - \pi_{ij}^{i,o} \pi_{ij}^{j,o} > 0 \). As in the case of PTS, output prices may be strategic complements (\( \pi_{ij}^{i,o} > 0 \)) or strategic substitutes (\( \pi_{ij}^{i,o} < 0 \)). Confining attention to first-order effects in expression (25), making substitutions from (18) and (19) yields

\[
\pi_{ij}^{i,o} \equiv \left( 1 - \frac{D_i^jS_j^i - D_i^jS_j^i}{\sum} \right)D_j - \left( \frac{D_j^iS_j^i - D_j^iS_j^i}{\sum} \right)D_i.
\]

In the symmetric market equilibrium, this can be written

\[
\pi_{ij}^{i,o} = \epsilon_d \left[ 1 + \frac{(p - w)}{w}e_{ij}^s (1 - \epsilon^s) \right] - \Theta,
\]

where the first term on the right-hand side of the expression is positive and the second term is negative under PTO (\( \Theta > 0 \)). The relative degree of product differentiation influences the mode of competition under PTO in the opposite manner as under PTS: \( \Theta > 0 \) is a necessary but not a sufficient condition for prices to be strategic substitutes under PTO.
The mode of competition in the linear case can be characterized as follows:

**PROPOSITION 4:** For the linear model, prices are strategic substitutes in the Pareto dominant equilibrium when:

(i) \( \epsilon_s \leq \frac{d}{1 + \frac{d}{2} \epsilon_s^2 (1- \epsilon_s^2)} \); or 
(ii) \( \epsilon_d \leq \frac{d}{1 + \frac{d}{2} \epsilon_d^2 (1- \epsilon_d^2)} \).

Under circumstances in which \( \Theta < 0 (\epsilon_s < \epsilon_d) \), the products in the upstream market are relatively more differentiated than products in the downstream market. Firms orient their pricing strategies towards the upstream market by anchoring input prices under PTS. Part (i) of Proposition 4 is the relevant criteria for the mode of competition, and prices are strategic substitutes when \( \epsilon_s \) is sufficiently small relative to \( \epsilon_d \). When \( \Theta > 0 (\epsilon_d < \epsilon_s) \), the products in the downstream market are relatively more differentiated and part (ii) of Proposition 4 is the relevant criteria for the mode of competition under PTO. Prices are strategic substitutes when \( \epsilon_d \) is sufficiently small relative to \( \epsilon_s \). In either case, the mode of competition is determined by the relative degree of product differentiation (i.e., the magnitude of \(|\Theta| > 0\)).

**III. Linear Example**

Consider the linear specialization of the model,

\[
D^i(p_1, p_2) = a - bp_i + cp_j,
\]

\[
S^i(w_1, w_2) = \beta w_i - \gamma w_j,
\]

where \( i = 1, 2, i \neq j \), and where \( a, b, \beta > 0 \) and \( c, \gamma \geq 0 \) are positive constants. Demand (supply) conditions reduce to local monopoly (monopsony) markets when \( c = 0 \) (\( \gamma = 0 \)) and products in the downstream (upstream) market are nearly homogeneous when \( c \rightarrow 0 (\gamma \rightarrow 0) \). 

We select parameters for the baseline calibration of the model such that \( \Theta = 0 \) (\( b\gamma = c\beta \)). This ensures that no differences exist in the equilibrium outcomes be-
tween Bertrand, PTO, and PTS in the baseline calibration. Specifically, we choose \( a = 1, b = 1, c = 0.5; \beta = 1, \gamma = 0.5, F = 0 \) for the baseline, which results in positive outcomes for profits \( (\pi^* = 0.222) \) and market power \( (L^* = 0.5) \).

Table 1 reports the outcomes for market power under PTO, PTS, Bertrand, and Cournot for variations in \( c \). The Pareto dominant equilibrium is PTO when the downstream market is relatively more differentiated than the upstream market \( (c < 0.5) \) and PTS when the upstream market is relatively more differentiated than the downstream market \( (c > 0.5) \). Prices are strategic complements when the degree of product differentiation in the downstream market satisfies \( 0.188 < c < 0.886 \) and are otherwise strategic substitutes.

The entries in Table 1 indicate that the degree of market power exercised by oligopoly firms follows a non-monotonic pattern with the degree of product differentiation in the downstream market. Market power decreases with \( c \) under PTO for values below \( c = 0.5 \), but market power increases with \( c \) under PTS thereafter. In the Bertrand equilibrium, market power is invariant to changes in \( c \), while market power rises monotonically in \( c \) under Cournot. The PTO equilibrium reduces to the Cournot outcome in the case of monopoly downstream markets \( (c = 0) \) and the PTS equilibrium converges to the Cournot equilibrium as the degree of product differentiation vanishes in the downstream market \( (c \to 1) \).

<table>
<thead>
<tr>
<th>Variation Output Differentiation</th>
<th>Market Power ( (L) )</th>
<th>Mode of Competition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c = 0.00 )</td>
<td>0.538</td>
<td>– 0.500 0.538</td>
</tr>
<tr>
<td>( c = 0.25 )</td>
<td>0.520</td>
<td>– 0.500 0.540</td>
</tr>
<tr>
<td>( c = 0.50 )</td>
<td>0.500</td>
<td>0.500 0.500 0.570</td>
</tr>
<tr>
<td>( c = 0.75 )</td>
<td>– 0.548</td>
<td>0.500 0.644 0.643</td>
</tr>
<tr>
<td>( c = 0.99 )</td>
<td>– 0.929</td>
<td>0.500 0.963 -11.435</td>
</tr>
</tbody>
</table>

Table 2 reports the outcomes for market power under PTO, PTS, Bertrand, and Cournot for variations in \( \gamma \). The Pareto dominant equilibrium is PTS when the upstream market is relatively more differentiated than the downstream market \( (\gamma < 0.5) \) and PTO when the
downstream market is relatively more differentiated than the upstream market \((\gamma > 0.5)\). Prices are strategic complements for values of \(\gamma\) that satisfy \(0.188 < \gamma < 0.886\) and are otherwise strategic substitutes.

The entries in Table 2 indicate that market power decreases monotonically in all cases as the degree of product differentiation declines in the upstream market (i.e., \(\gamma\) increases). The oligopoly equilibrium reduces to Cournot in the case of monopsony input markets \((\gamma = 0)\).

### Table 2 – Market Power for Variations in Upstream Product Differentiation

<table>
<thead>
<tr>
<th>Variation Wholesale Differentiation</th>
<th>Market Power ((L))</th>
<th>Mode of Competition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma = 0.00)</td>
<td>PTO 0.700</td>
<td>PTS 0.667</td>
</tr>
<tr>
<td>(\gamma = 0.25)</td>
<td>PTO 0.619</td>
<td>PTS 0.600</td>
</tr>
<tr>
<td>(\gamma = 0.50)</td>
<td>PTO 0.500</td>
<td>PTS 0.500</td>
</tr>
<tr>
<td>(\gamma = 0.75)</td>
<td>PTO 0.377</td>
<td>PTS 0.333</td>
</tr>
<tr>
<td>(\gamma = 0.99)</td>
<td>PTO 0.208</td>
<td>PTS 0.019</td>
</tr>
</tbody>
</table>

For variation in the relative degree of product differentiation, the mode of competition switches from strategic complements to strategic substitutes under both PTS and PTO as the degree of oligopoly interaction becomes less balanced across the upstream and downstream markets. In the subsequent section we use our model to describe the circumstances that favor various forms of strategic pre-commitment.

### IV. Strategic Pre-Commitment

In this section, we derive policy implications of the model under three common forms of strategic delegation: \((i)\) international trade; \((ii)\) vertical separation; and \((iii)\) managerial incentives.

Suppose a principal (a domestic trade authority, a firm, or a controlling shareholder) writes an observable and enforceable contract with her agent (a domestic firm, a supplier/consumer, or a manager) with the goal of increasing the profit of firm \(i\). The compensation in the agent’s contract takes the form \(A_i + B_i M_i\), where \(A_i\) and \(B_i\) are constants
specified in the contract and

\[ M^i = \alpha_i \pi^i + ((1 - \alpha_i) p_i - t_i) y_i \]

contains the values \((\alpha_i, t_i)\) that are strategically relevant.\(^7\) This contract accords with a managerial contract that involves a weighted average of sales and profits when \(t_i = 0\), and with the profit function of a firm engaging in a two-part tariff contract with suppliers (or consumers) as well as with the profit function of a domestic firm facing a unit tax on a traded good when \(\alpha_i = 1\).

Our game now consists of three stages. In the first stage, principal \(i, i = 1, 2\), proposes a contract to her agent of the form in equation (26). In the second-stage, agents select the mode of competition (PTO or PTS), with competition in prices in one market resulting in market-clearing outcomes in the remaining market in the final stage of the game.

Consider first the case of PTS. In the event that the contract is accepted, agent \(i\)’s problem when facing the contract specified in (26) is to maximize

\[ M^{i,s}(w_i, w_j, \alpha_i, t_i, F_i) = \left( p^i(w) - \alpha_i w_i - t_i \right) S^i(w) - F_i. \]

The first-order necessary condition for a maximum is

\[ (27) \quad M^{i,s}_{ii} = \left( p^i(w) - \alpha_i w_i - t_i \right) S^i_{ii}(w) + \left( p^i_{ii}(w) - \alpha_i \right) S^i_{i}(w) = 0, \quad i = 1, 2. \]

Confining attention to first-order effects, the second partials of \(M^{i,s}\) with respect to \(w_i, w_j\) are

\[ M^{i,s}_{ii} = 2 \left( p^i_{ii}(w) - \alpha_i \right) S^i_{i}(w) < 0, \]

\[ M^{i,s}_{ij} = p^i_{ij}(w) S^i_{i}(w) + \left( p^i_{i}(w) - \alpha_i \right) S^i_{j}(w). \]

\(^7\)When faced with this contract, the agent’s problem is equivalent to maximizing \(M^i\).
The effect of the policy variables on the agent’s first order condition (27) are

\[ M_{i,s}^i = - \left( S^i(w) + w_i S^i_i(w) \right) < 0 \]

\[ M_{i,t}^i = - S^i_i(w) < 0. \]

By the implicit function theorem, we have

\[ \frac{\partial w_j}{\partial w_i}(w_1, w_2) = \frac{\partial w_j/\partial \alpha_i}{\partial w_i/\partial \alpha_i} = \frac{\partial w_j/\partial t_i}{\partial w_i/\partial t_i} = \frac{-M_{ji}^{i,s}}{M_{jj}^{i,s}}. \]

Equation (28), which defines the slope of the rival’s reaction function, takes the sign of \( M_{ji}^{i,s} \).

Now consider the case of PTO. In the case where the contract is accepted, agent \( i \)’s problem when posed with contract specified in (26) is to maximize

\[ M^{i,o}(p_i, p_j, \alpha_i, t_i, F_i) = \left( p_i - \alpha_i w^i(p) - t_i \right) D^i(p) - F_i, \]

The first-order necessary condition for a maximum is

\[ (29) \quad M_{i,o}^{i,o} = (p_i - \alpha_i w^i(p) - t_i) D^i(p) + (1 - \alpha_i w^i(p)) D^i(p) = 0, \quad i = 1, 2. \]

Confining attention to first-order effects, the second partials of \( M^{i,o} \) with respect to \( p_i, p_j \), are:

\[ M_{ii}^{i,o} = 2D^i_i(p) \left( 1 - \alpha_i w^i(p) \right) < 0, \]

\[ M_{ij}^{i,o} = (1 - \alpha_i w^i(p)) D^i_j(p) - \alpha_i w^i_j(p) D^i_j(p). \]

Proceeding as above, the slope of the rival’s reaction function is

\[ (30) \quad \frac{\partial p^j}{\partial p^i}(p_1^i, p_2^i) = \frac{\partial p^j_i/\partial \alpha_i}{\partial p_i^i/\partial \alpha_i} = \frac{\partial p^j_i/\partial t_i}{\partial p_i^i/\partial t_i} = \frac{-M_{ji}^{i,o}}{M_{ij}^{i,o}} = M_{ji}^{i,o}. \]

In the second stage game, the agent chooses between PTS or PTO according to the
terms of the contract. Analysis of this decision reveals

**PROPOSITION 5:** The choice of PTO or PTS by an agent is invariant to the principal’s choice of policy variables \((\alpha_i, t_i)\). Thus, the Pareto dominant equilibrium is Bertrand if \(\Theta = 0\), PTO if \(\Theta > 0\), and PTS if \(\Theta < 0\).

The magnitudes of the policy variables \((\alpha_i, t_i)\) influence the value of \(\Theta\), but not the sign. An implication of Proposition 5 is that our observations on strategic pre-commitment devices are robust to different policy structures that alter the timing of the game.

Next consider the problem of the agent under PTS. Letting \(D = (\alpha_1, \alpha_2)\) and \(t = (t_1, t_2)\) denote the vectors of policy variables, profit is

\[
\tilde{\pi}^{i,s}(\alpha, t) = (p^j(w(\alpha, t)) - w_i(\alpha, t)) S^j(w(\alpha, t)).
\]

Dropping arguments for expositional convenience, the first-order condition with respect to policy variable \(\delta_i = (\alpha_i, t_i)\) is

\[
(31) \quad \tilde{\pi}^{i,s}_i \equiv [(p^i - w_i) S^i_i + (p^i - 1)S^i_i] \frac{\partial w_i}{\partial \delta_i} - [(p^i - w_i) S^i_j + p^j S^j_i] \frac{\partial w_j}{\partial \delta_i} = 0.
\]

Making use of the agent’s first-order condition (27), equation (31) can be rearranged as

\[
t_i S^i_i - (1 - \alpha_i)(w_i S^i_i + S^i) = - [(p^i - w_i) S^i_j + p^j S^j_i] \frac{\partial w_j}{\partial w_i}.
\]

Notice that the term in square brackets on the right-hand side of this equation is negative whenever PTS is a Pareto dominant strategy, because \(p^j(w) < 0\) under PTS. It follows from expression (28) that

\[
(32) \quad t_i S^i_i - (1 - \alpha_i)(w_i S^i_i + S^i) = M^{i,s}_{ji}.
\]

Notice that the optimal choice of policy variables for principal \(i\), \(\delta_i = (\alpha_i, t_i)\), depends on the mode of competition. For \(\alpha_i = 1\), \(t_i = M^{i,s}_{ji}\), and for \(t_i = 0\), \(\alpha_i - 1 = M^{i,s}_{ji}\).
Under PTO,

$$\tilde{\pi}^{i,o}(\alpha, t) = (p_i(\alpha, t) - w^i(p(\alpha, t))) D^i(p(\alpha, t)).$$

The first-order condition for a profit maximum is

$$\tilde{\pi}^{i,o}_i = \left[ (p_i - w^i) D^i_i + (1 - w^i) D^i_j \right] \frac{\partial p_i}{\partial \delta_i} + \left[ (p_i - w^i) D^i_j - w^i D^i_j \right] \frac{\partial p_j}{\partial \delta_i} = 0.$$

Making use of the firms's first-order condition \((29), (33)\) can be rearranged as

$$(1 - \alpha_i)(w^i D^i_i + w^j D^i_j) - t_i D^i_i = \left[ (p_i - w^i) D^i_j - w^i D^i_j \right] \frac{\partial p_j}{\partial p_i}.$$

The term in square brackets on the right-hand side of this equation is positive whenever PTO is a Pareto dominant strategy (i.e., \(w^j < 0\)). Hence, by expression \((30)\) the optimal contract by principal \(i\) satisfies

$$\frac{1}{1 - \alpha_i} (w^i D^i_i + w^j D^i_j) - t_i D^i_i = M^{i,o}_{ji}.$$

Notice that the implication of the mode of competition for strategic pre-commitment policy is identical under PTS and PTO. For \(\alpha_i = 1, t_i = M^{i,o}_{ji}, \) and for \(t_i = 0, \alpha_i = 1 \equiv M^{j,o}_{ji}.\)

### A. Strategic Trade

The implications of our model for strategic international trade depend on the orientation of the markets. To see this, consider first an extension of the usual “third country” setting in which the upstream and downstream markets are located in a third and fourth country, respectively. The “third country” assumption is convenient for isolating the strategic rent-shifting implications of pre-commitment policy in the oligopoly model, and the location of the input and output markets outside the producing countries is a natural extension of the strategic trade framework to intermediated oligopoly.
The optimal strategic trade policy can be examined by setting $a_i = 1$ in equations (32) and (34). Under PTS, the role of strategic trade policy is the usual one of rent shifting in the downstream international output market and the optimal strategic trade policy is a subsidy when $M_{ji} = \pi_{ji} < 0$ and a tax when $\pi_{ji} > 0$. Export subsidies emerge as a strategic trade policy when products in the upstream market are relatively highly differentiated, for instance when the international downstream market is characterized by nearly homogeneous goods. Under circumstances of relatively equal levels of product differentiation in upstream and downstream markets, taxes are optimal as a strategic trade policy; however, the practical way to implement such a policy would be to levy a tariff on imports from the upstream international market. Our intermediated oligopoly model encompasses the use of imports tariffs as a rent-shifting policy instrument, which can substitute for the usually-suggested policy of export taxes in the strategic trade literature. Indeed, tariffs on imported inputs are commonly observed in developed countries – much more so than export taxes.

Under PTO, the role of strategic trade policy is to shift rents in the upstream international input market. The optimal strategic trade policy is a subsidy when $M_{ji} = \pi_{ji} < 0$, and tax when $\pi_{ji} > 0$. Export subsidies are optimal as a strategic trade policy when products in the upstream market are relatively highly differentiated, which can occur when the international upstream market is characterized by nearly homogeneous goods (e.g., commoditized components used for consumer electronics). Under circumstances where taxes are optimal as a strategic trade policy, the natural policy would again be a tariff on imports in the upstream market.

Now consider the case where at least a portion of consumers and/or input suppliers reside in the domestic country. The location of input suppliers and consumers in the domestic country increases the incentive to levy export subsidies, because export subsidies also serve in this case to reduce the oligopoly distortion in the markets served by domestic suppliers and consumers. Suppose the upstream markets are segmented across the two producing countries, for instance when a domestic farm input is used to produce a traded agricultural product. With a domestic upstream market, the supply conditions
facing the domestic firm in the upstream market are uncorrelated with the supply conditions facing the foreign rival, and PTS is the Pareto dominant strategy. The mode of competition reduces to Cournot oligopoly, and export subsidies are optimal as a strategic trade policy, as in Brander and Spencer (1985).

Now suppose that the downstream markets are segmented across the two producing countries. It follows that PTO is the Pareto dominant strategy. The oligopoly outcome reduces to Cournot oligopsony and subsidies are optimal as a strategic trade policy to shift rents in the international upstream market. Given that trade does not occur in downstream consumer goods, the domestic firm would receive a subsidy on foreign input procurement (a negative tariff).

Finally, note that the only case in which an import tariff would not be available as a strategic trade policy is the one in which the upstream market is located entirely in the domestic country. However, the role of export taxes here would be quite different from the usual rent-shifting motivation, as foreclosure of the input market to the foreign rival would now be possible. The optimal policy generally would be an export tax in the upstream market, which would serve to selectively raise the rival’s cost.

The intermediation model allows the optimal value of the strategic pre-commitment policy to be defined according to supply and demand conditions facing firms. We illustrate this property for the case of linear supply and demand functions by calculating the (unique) Nash equilibrium in export policy variables for the case of \( a_i = 1 \) in equations (32) and (34). Specifically, we consider variation in the degree of downstream product differentiation \( c \) at baseline values of the remaining parameters in the model (\( a = 1, b = 1, \beta = 1, \gamma = 0.5, F = 0 \)) and identify the optimal policy in the symmetric pre-commitment equilibrium.

Figure IV.A depicts the optimal value of \( t^* = t^*_i = t^*_j \) in the symmetric Nash policy equilibrium in response to variations in \( c \). The optimal policy is a subsidy \( (t^* < 0) \) for levels of product differentiation that satisfy \( c < 0.188 \) and \( c > 0.886 \) and is otherwise a tax \( (t^* > 0) \). An inverted u-shaped pattern emerges for the optimal policy variable.
B. Vertical Separation

Now consider the optimal policy of an intermediary that seeks to vertically separate production with a two-part tariff contract. The optimal policy for vertical separation can be examined by setting $\alpha_i = 1$ in equations (32) and (34). The optimal contract with suppliers in the upstream market involves $t_i > 0$ and a lump-sum transfer paid to the retailer (a “slotting allowance”) under PTS (PTO) when $\pi_{ji}^{i,s} (\pi_{ji}^{i,o}) > 0$. Slotting allowances are optimal for products with relatively equal degree of product differentiation in the upstream and downstream markets.

When $\pi_{ji}^{i,s} < 0$ or $\pi_{ji}^{i,o} < 0$, the optimal contract involves $t_i < 0$. This contract can be implemented in the upstream market by paying suppliers a lump-sum transfer (a negative slotting allowance) in exchange for receiving a lower wholesale price. The contract can also be implemented in the downstream market by charging membership fees to consumers in exchange for paying lower retail prices. Membership fees are common in some retail environments, for instance at discount warehouses like Costco and Sams Club, and the intermediation model provides an explanation for this phenomenon as a
strategic rent-shifting device.

C. Managerial Incentives

Now consider the optimal policy of an agent who selects a managerial contract. The optimal managerial contract can be examined by setting \( t_i = 0 \) in equations (32) and (34).

Note that \( \alpha_i \) changes the slope of the reaction function facing the rival, \( -\frac{M_{ij}^{\alpha}}{M_{ij}^{\beta}} \) and \( -\frac{M_{ij}^{\alpha}}{M_{ij}^{\beta}} \) under PTS and PTO, respectively. The change in the slope of the rival’s reaction function when the rival principal selects terms of her managerial contract can alter the magnitude of the optimal policy variable. Nevertheless, at least for the linear case, the equilibrium selection of policy variable does not alter the market conditions that favor over-emphasizing sales or profits.

Figure IV.C depicts the optimal value of \( \alpha^* = \alpha_i^* = \alpha_j^* \) in the symmetric Nash policy equilibrium in response to variations in \( c \). The optimal policy over-emphasizes sales (\( \alpha^* < 1 \)) for levels of product differentiation that satisfy \( c < 0.188 \) and \( c > 0.886 \) and over-emphasizes profits (\( \alpha^* > 1 \)) for levels of product differentiation that satisfy \( 0.188 < c < 0.886 \). An inverted u-shaped pattern emerges for the optimal policy variable as in the case of a unit tax (subsidy) scheme.
Optimal managerial incentive \((\alpha)\) for variations in downstream market differentiation.

V. Concluding Remarks

Our model results in testable hypotheses on the mode of competition that can be used to derive policy implications under oligopoly. The prevailing mode of competition in a particular industry is an empirical question in the intermediated oligopoly model that depends on the relative degree of product differentiation in the upstream and downstream markets.

The oligopoly intermediation model allows the mode of competition to be characterized quite precisely according to commonly-estimated features of supply and demand functions. We demonstrated that prices are strategic complements when the strength of the oligopoly interaction is relatively balanced in the upstream market and the downstream market that serve as the points of contact for firms. In both the PTS and PTO equilibrium, prices change from strategic complements to strategic substitutes as the relative degree of product differentiation between markets becomes sufficiently high. The mode of competition in a particular industry, and the commensurate sign and magnitude of the optimal pre-commitment policy, therefore depends entirely on primitives of supply
and demand functions facing firms.

Our model provides empirical direction for examining the mode of competition in industrial settings. Empirical research is thus capable of tailoring pre-commitment policies to the conditions facing firms in individual industries, which can provide a useful tool for policy formulation as well as for anti-trust regulation.

An interesting avenue for future research is to consider an active role for firms in contributing to product differentiation. The more firms contribute to product differentiation at their stage of production, whether by creating innovative new products from existing inputs or by finding new ways to produce existing products from specialized inputs, the more likely their prices are to be strategic substitutes. It is clear from our analysis that relaxing the degree of oligopoly interaction with rivals is not a globally desirable endeavor, and the reason is that it is the relative degree of product differentiation, and not the absolute degree, that is the source of value-added in a vertical structure.

Another potentially fruitful extension of our model is to consider multiple inputs in the production functions of firms. With more than one upstream market providing inputs to oligopoly firms, we would expect the same basic insights described here to emerge for a given technology in terms of a composite input requirement; however, if technology is endogenous, biased technologic change may be desirable for strategic reasons. Biased technological change can allow firms to emphasize input procurement from more (or less) differentiated markets. Given that it is the relative degree of product differentiation in the upstream and downstream markets that contributes to market power, firms may find it desirable to emphasize the relative degree of product differentiation in the downstream market. Thus, our model seems capable of providing a strategic explanation for technology standardization under circumstances where products in the downstream market are sufficiently differentiated.

REFERENCES


MATHEMATICAL APPENDIX

In this Appendix we derive the proofs of all propositions. Propositions 1 and 4 hold by inspection.

Proof of Proposition 2.

The proof is constructed by comparing outcomes under PTO, PTS, and Bertrand-Nash to the monopoly outcome. Given that the profit function of each firm is concave (convex) in $p(w)$, our goal is to show that the monopoly prices $(p^M, w^M)$ satisfy $w^M < w^o < w^B$ and $p^M > p^o > p^B$. When this condition holds, firm profits are rising for defections from $w^B (p^B)$ that involve $w < w^B (p > p^B)$ under PTS (PTO).
A monopoly firm chooses a vector of quantities, $y = (y_1, y_2)$, to maximize profits of

$$
\pi^M(y) = \sum_{i=1,2} \left[ \left( P^i(y) - W^i(y) \right) y_i - F_i \right],
$$

where $P^i(y)$ is inverse demand and $W^i(y)$ is inverse supply for product $i$. The first-order necessary condition is

$$
(A1) \quad \pi^M_i(y) \equiv P^i - W^i + \left( \frac{\partial P^i}{\partial y_i} - \frac{\partial W^i}{\partial y_i} \right) y_i + \left( \frac{\partial P^j}{\partial y_i} - \frac{\partial W^j}{\partial y_i} \right) y_j = 0, \quad i = 1, 2, i \neq j
$$

Simultaneously solving (A1) in the case of a symmetric equilibrium gives the equilibrium quantity, $y^M$, which can be used to recover the monopoly equilibrium prices.

To facilitate the comparison of the monopoly outcome with the Bertrand, PTS, and PTO equilibria, note from (11) and (12) that

$$
\frac{\partial P^i}{\partial y_i} = \frac{D^i_j}{\Delta} < 0 \quad \text{and} \quad \frac{\partial W^i}{\partial y_i} = \frac{-D^i_j}{\Delta} < 0.
$$

Similarly, it is straightforward to verify that

$$
\frac{\partial W^i}{\partial y_i} = \frac{S^j_i}{\Xi} > 0 \quad \text{and} \quad \frac{\partial W^j}{\partial y_i} = \frac{-S^j_i}{\Xi} > 0.
$$

Incorporating these terms in (A1) and evaluating the slope of the monopoly profit function at the Bertrand price level, $(p^B - w^B) = \left( \frac{S^j_i}{\Xi} - \frac{D^i_j}{\Delta} \right)$, gives (after slight manipulation)

$$
\pi^M_i(p^B, w^B) = y \left[ \frac{S^j_i (S^j_i - S^j_j)}{S^j_i \Xi} + \frac{D^j_i (D^j_i - D^j_j)}{D^j_i \Delta} \right] < 0,
$$

where the sign holds by the stability condition. Hence the symmetric monopoly output level satisfies $y^M < y^B$. By the concavity of the profit expression, $w^M < w^B$ and $p^M > p^B$.

It remains to compare the monopoly solution to the equilibrium outcomes under PTS and PTO. Our conjecture is that profitable defections from the Bertrand-Nash equilibrium involve $w < w^B$ and $p > p^B$; hence, firms play PTO if $\Theta > 0$ and firms play PTS if $\Theta < 0$. This conjecture is correct provided that $w^M < w^s < w^B$ and $p^M > p^o > p^B$. To verify this, consider the slope of monopoly profit at the symmetric PTS equilibrium
position, which is

\[ \pi^M_i(p^s, w^s) = \frac{y}{S^i_j} \left[ 1 - \frac{(D^i_j - D^i_j)}{\Delta} \right] + y \left( \frac{D^i_j}{\Delta} - \frac{S^i_j}{\Sigma} - \frac{D^i_j}{\Sigma} + \frac{S^i_j}{\Sigma} \right). \]

Factoring this expression yields

\[ \pi^M_i(p^s, w^s) = \frac{y \left( S^i_j - S^j_i \right)}{S^i_j} \left( \frac{S^i_j}{\Sigma} - \frac{D^i_j}{\Delta} \right) < 0, \]

where the sign holds by the stability condition. The symmetric monopoly output level satisfies \( y^M < y^s \), and it follows by the concavity of the profit expression that \( w^M < w^s \) and \( p^M > p^s \).

Proceeding similarly in the case of PTO competition, evaluating the slope of monopoly profit at the symmetric PTO equilibrium position gives

\[ \pi^M_i(p^o, w^o) = \frac{y \left( D^i_j - D^j_i \right)}{D^i_j} \left( \frac{S^i_j}{\Sigma} - \frac{D^i_j}{\Delta} \right). \]

By inspection, \( y^M < y^o \), and it follows that \( w^M < w^o \) and \( p^M > p^o \) under PTO.

**Proof of Proposition 3.**

First consider PTS. Note from equation (11) and the associated condition on the supply-side that \( \frac{\partial P^i}{\partial y^i} = \frac{D^i_j}{\Delta} < 0 \) and \( \frac{\partial P^i}{\partial y^j} = \frac{S^i_j}{\Sigma} > 0 \). Incorporating these terms in the optimality condition, and making use of the resulting equation to evaluate the slope of profit expression (15) at the symmetric Cournot equilibrium position \((w^C, p^C)\) gives

\[ \pi^i_s(w^C, p^C) = S^i_j \left( \frac{S^i_j}{\Sigma} - \frac{D^i_j}{\Delta} \right) \leq 0. \]

The term in brackets is negative by our stability conditions. By inspection, the Cournot outcome emerges under PTS \((y^s = y^C)\) when \( S^i_j = 0 \) \((\epsilon_s = 0)\).
Proceeding similarly in the case of PTO competition, evaluating the slope of the profit expression (20) at the symmetric Cournot equilibrium position \((w^C, p^C)\) gives

\[
\pi_{i}^{i,o}(w^C, p^C) = D_i D_j \left( \frac{S^i_j}{\Delta} - \frac{D^i_j}{\Delta} \right) \leq 0.
\]

By inspection, \(y_a = y^C\) when \(D_j^i = 0\) \((\epsilon_d = 0)\).

**Proof of Proposition 5.**

The proof is constructed by considering price movements from the Bertrand equilibrium position under PTS and PTO. In the case of an interior solution, the Bertrand equilibrium for an agent facing a contract of the form in (26) is defined by the first-order necessary condition

\[
p_i - a_i w_i - t_i = a_i \frac{S^i_i(w)}{S^i_i(w)} - \frac{D^i_i(p)}{D^i_i(p)}, \quad i = 1, 2
\]

Under PTS competition, evaluating the agent’s input price condition (27) at the symmetric Bertrand equilibrium position \((w^B, p^B)\) gives

\(\text{(A2)} \quad M_{i}^{i,s}(w^B, p^B) \equiv \frac{S_i^j D_i^j S_j^i}{\Delta} \Theta \doteq \Theta.\)

Under PTS competition, evaluating the output price condition (20) at the symmetric Bertrand equilibrium position \((w^B, p^B)\) gives

\(\text{(A3)} \quad M_{i}^{i,o}(w^B, p^B) \equiv \frac{a_i D^i_i D^j_j S^j_j}{\Sigma} \Theta \doteq \Theta.\)

By Proposition 2, oligopoly profits rise with defections from the Bertrand equilibrium \((p', w')\) that involve \(p' > p^B\) and \(w' < w^B\). The remainder of the proof follows immediately by inspection of terms in equations (A2) and (A3).